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Simultaneous Mutual Interference and Jamming  
in a Frequency-Hopping Network

by Don J. Torrieri

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U.S. Army Materiel Development  
and Readiness Command

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## 1. THE BASIC MODEL

The effects of various types of jamming on a frequency-hopping receiver have been previously examined.<sup>1</sup> However, frequency-hopping communicators do not often operate in isolation. Instead, they are usually elements of a network of frequency-hopping systems that cause mutual interference. It is important to assess how a system's performance is affected by the combination of simultaneous mutual interference and jamming. We consider a network of frequency-hopping systems that have omnidirectional antennas, generate the same output power, share the same  $M$  frequency channels, and are nearly stationary over a bit duration.

Frequency hopping is the periodic changing of the frequency or frequency sets associated with a transmission. If the data modulation is multiple frequency-shift keying, two or more frequencies are in the set that changes at each hop. For other data modulations, a single frequency is changed at each hop. We shall consider data modulations such that sets of one or two frequencies change at each hop. The generalization to sets of more than two frequencies is straightforward.

We initially neglect spectral splatter and inter-modulation products; that is, interference from other hoppers occurs in a channel only if at least one of the other hoppers is using this channel as its transmission channel. Effects due to differences in hopping transition times throughout the network are ignored.

Consider the transmission of a bit from a hopper at A to a receiver at B, as depicted in figure 1. The distance between the two is  $D_c$ . The light dots in the figure represent some of the  $N$  potentially interfering hoppers in the network of  $N + 2$  total hoppers. Each interferer is labeled by an index  $i$ .

We initially assume that the data modulation is binary frequency-shift keying (FSK). This modulation requires that a pair of frequency channels are

associated with each transmitted bit. As explained elsewhere,<sup>1</sup> the spectrum occupied by a transmitted bit is called the transmission channel. The spectrum that would be occupied if the logical state represented by the bit were reversed is called the complementary channel. Both channels change with the frequency hops.

The event  $A_{jk}$  is the event that  $j$  of the interferers use the transmission channel and  $k$  of the interferers use the complementary channel during the reception of a desired bit at B. We denote the probability of  $A_{jk}$  by  $P(A_{jk})$ . The probability of a bit error given  $A_{jk}$  is denoted by  $P_{jk}$ . Since  $P(A_{jk}) = 0$  if  $j + k > N$ , the probability of a bit error is

$$P_b = \sum_{j=0}^N \sum_{k=0}^{N-j} P(A_{jk}) P_{jk} \quad (1)$$

The probability that power from an interferer enters the transmission channel is  $d/M$ , where  $d$  is the duty factor, that is, the probability that the interferer is emitting power during a bit interval. Similarly, the probability that the power from an interferer enters the complementary channel is  $d/M$ . The probability that the power does not enter either channel is  $1 - 2d/M$ . There are

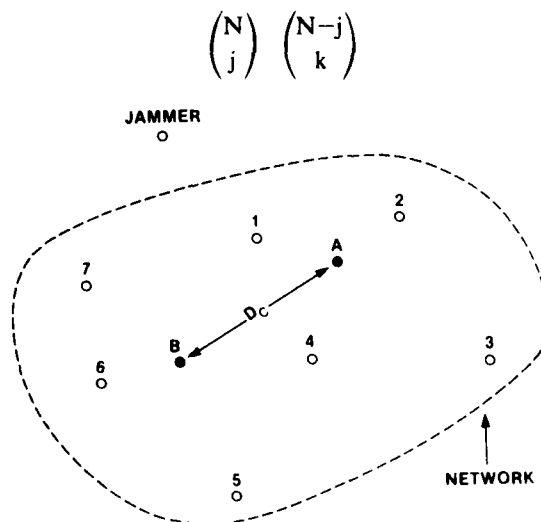


Figure 1. Frequency-hopping network and jammer.

<sup>1</sup>D. J. Torrieri, *Frequency Hopping in a Jamming Environment*, U.S. Army DARC/MCM CCM 7R.2 (December 1978)

ways to select one set of  $j$  (transmission channel) interferers and another set of  $k$  (complementary channel) interferers when  $j + k \leq N$ . Thus,

$$P(A_{jk}) = \left(\frac{d}{M}\right)^j \left(\frac{d}{M}\right)^k \left(1 - \frac{2d}{M}\right)^{N-j-k} \times \binom{N}{j} \binom{N-j}{k} \quad (2)$$

Substituting into equation (1) yields

$$P_b = \sum_{j=0}^N \sum_{k=0}^{N-j} \left(1 - \frac{2d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} P_{jk} \quad (3)$$

If the deployment of interferers is specified statistically, each  $P_{jk}$  in the summation can be expressed as a multiple integral.<sup>2</sup> If the interferer  $i$  is using the transmission channel, the ratio of the power from interferer  $i$  to the power of the desired signal at the receiver is denoted by  $x_i$ . If interferer  $i$  is using the complementary channel, the ratio of the power from interferer  $i$  to the power of the desired signal at the receiver is denoted by  $y_i$ . Let  $P(x_1, \dots, x_j, y_1, \dots, y_k)$  denote the probability of a bit error given that  $x_1, x_2, \dots, x_j, y_1, y_2, \dots, y_k$  are the interference-to-signal ratios caused by  $j$  interferers that use the transmission channel and  $k$  interferers that use the complementary channel. Let  $f(u)$  denote the probability density function for an interference-to-signal ratio due to a single interferer. Since each interferer is located and hops independently of the other interferers, the probability density function for  $x_1, x_2, \dots, x_j, y_1, y_2, \dots, y_k$  given  $A_{jk}$  is

$$\prod_{i=1}^j f(x_i) \prod_{i=1}^k f(y_i)$$

Thus the definition of  $P_{jk}$  implies that

$$P_{jk} = \int_0^\infty \dots \int_0^\infty P(x_1, \dots, x_j, y_1, \dots, y_k) \prod_{i=1}^j f(x_i) \prod_{i=1}^k f(y_i) dx_1 \dots dx_j dy_1 \dots dy_k \quad (4)$$

In general, approximations of this expression are necessary to obtain an estimate of the bit error probability that is computationally reasonable. To avoid evaluating multiple integrals of order greater than  $L$ , we can set  $P_{jk} = 0$  for  $j + k > L$  in equation (3). This truncation gives a lower bound for  $P_b$  if  $N > L$ . Denoting this lower bound by  $P_L$ , we have

$$P_L = \sum_{\substack{j=0 \\ j+k \leq L}}^N \sum_{k=0}^{N-j} \left(1 - \frac{2d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} P_{jk} \quad (5)$$

If  $N > L$ , an upper bound for  $P_b$  results if we set  $P_{jk} = 1$  for  $j + k > L$ . The difference between the two bounds is  $P(j + k > L)$ , the probability that more than  $L$  interferers produce power in one of the two channels associated with the transmission of a bit. Denoting the upper bound for  $P_b$  by  $P_U$ , we have

$$P_U = P_L + P(j + k > L) \quad (6)$$

$$P(j + k > L) = \sum_{\substack{j=0 \\ j+k > L}}^N \sum_{k=0}^{N-j} \left(1 - \frac{2d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} \quad (7)$$

As  $dN/M$  decreases,  $P(j + k > L)$  decreases and the upper and lower bounds become tighter.

For single-channel data modulations, which use only one channel for the transmitted bits during

<sup>2</sup>S. A. Musa and W. Wasilikowski, Co Channel Interference of Spread Spectrum Systems in a Multiple User Environment, *IEEE Trans. Comm. COM-26* (October 1978), 1405.

a hopping period, we can derive simple analogous results. The probability of a bit error is

$$P_b = \sum_{j=0}^N \left(1 - \frac{d}{M}\right)^{N-j} \left(\frac{d}{M}\right)^j \binom{N}{j} P_j \quad (8)$$

where  $P_j$  is the probability of a bit error given that  $j$  interferers use the transmission channel. We can express  $P_j$  as the multiple integral

$$P_j = \int_0^\infty \dots \int_0^\infty P(x_1, \dots, x_j) \prod_{i=1}^j f(x_i) dx_1 \dots dx_j \quad (9)$$

where  $P(x_1, \dots, x_j)$  is the probability of a bit error given that  $x_1, \dots, x_j$  are the interference-to-signal ratios caused by  $j$  interferers using the transmission channel. The lower bound of  $P_b$  is

$$P_L = \sum_{j=0}^L \left(1 - \frac{d}{M}\right)^{N-j} \left(\frac{d}{M}\right)^j \binom{N}{j} P_j \quad (10)$$

For single-channel data modulations, we usually have  $P_j \leq 1/2$  so that

$$P_U = P_L + \frac{1}{2} P(j > L) \quad (11)$$

where

$$P(j > L) = \sum_{j=L+1}^N \left(1 - \frac{d}{M}\right)^{N-j} \left(\frac{d}{M}\right)^j \binom{N}{j} \quad (12)$$

## 2. DEPLOYMENT STATISTICS

Let  $r$  represent the distance between an interferer and the receiver at B in figure 1, and  $u$  represent the potential interference-to-signal ratio at the receiver. If  $g(r)$ , the radial density function for  $r$ , and a propagation model are specified, then

$f(u)$  can be determined. A reasonable approximate model for vhf ground-to-ground communications is that the received power varies inversely as the fourth power of the distance to the source. Thus, if the hoppers have identical system parameters, the interference-to-signal ratio at a receiver is

$$u = b \left(\frac{D_c}{r}\right)^4 = \left(\frac{D}{r}\right)^4 \quad (13)$$

where  $b$  is a proportionality constant that depends upon the details of the propagation law, such as the antenna heights, and the normalized communication distance is  $D = D_c b^{1/4}$ . Although we henceforth assume the validity of equation (13), it is straightforward to generalize the following analysis to obtain analogous results for received power varying inversely as an arbitrary power of the distance to the source.

A plausible statical deployment model is illustrated in figure 2. The receiver is at point B, which is the center of two circles with radii  $R_0$  and  $R_1$ , such that  $R_1 \geq R_0 \geq 0$ . The intended transmitter is at point A, a distance  $D_c$  from the receiver. The interferers are assumed to be uniformly distributed in the annular ring between  $R_0$  and  $R_1$ . Radius  $R_0$  is the minimum possible separation between an interferer and the receiver. To be consistent with the previous assumption that spectral splatter is insignificant,  $R_0$  must usually be greater than some minimum distance that is a function of  $D_c$  and the spectral splatter. Radius  $R_1$  is the maximum possible separation between the receiver and a significant interferer. In other words, if an interferer is at a greater distance than  $R_1$  from the receiver, the interferer contributes negligibly to the bit error rate. The radial density corresponding to figure 2 is

$$g(r) = \begin{cases} \left(\frac{2}{R_1^2 - R_0^2}\right) r & R_0 \leq r \leq R_1 \\ 0 & r < R_0, r > R_1 \end{cases} \quad (14)$$

which is depicted in figure 3. Elementary probability theory and equations (13) and (14) give

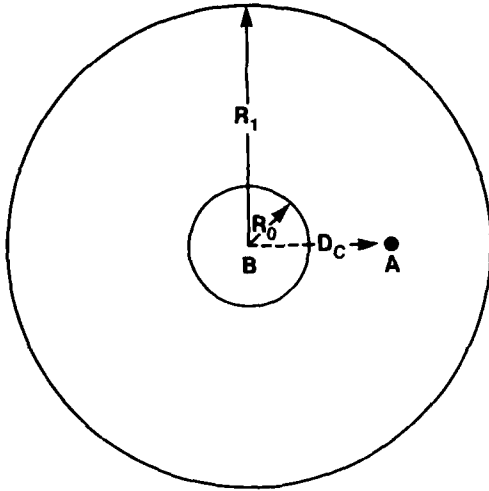


Figure 2. Geometry of uniform deployment.

$$f(u) = \begin{cases} \left[ \frac{D^2}{2(R_1^2 - R_0^2)} \right] u^{3/2}, & \left( \frac{D}{R_1} \right)^4 \leq u \leq \left( \frac{D}{R_0} \right)^4 \\ 0 & , u < \left( \frac{D}{R_1} \right)^4, u > \left( \frac{D}{R_0} \right)^4 \end{cases} \quad (15)$$

Musa and Wasylkiwskyj<sup>2</sup> have proposed a particular radial density that makes it possible to express  $P_{jk}$  as a double integral for all  $j$  and  $k$  and  $P_j$  as a single integral for all  $j$ . The radial density is

$$g(r) = \begin{cases} \frac{5\beta^4}{r^5} \exp \left[ -\frac{5}{4} \left( \frac{\beta}{r} \right)^4 \right] & , r > 0 \\ 0 & , r \leq 0 \end{cases} \quad (16)$$

The parameter  $\beta$  is the value of  $r$  at which  $g(r)$  is a maximum. It is the radial distance from the receiver at which an interferer is most likely to be found. Elementary probability theory and equations (13) and (16) give

<sup>2</sup>S. A. Musa and W. Wasylkiwskyj, Co Channel Interference of Spread Spectrum Systems in a Multiple User Environment, IEEE Trans. Comm. COM 26 (October 1978), 1405.

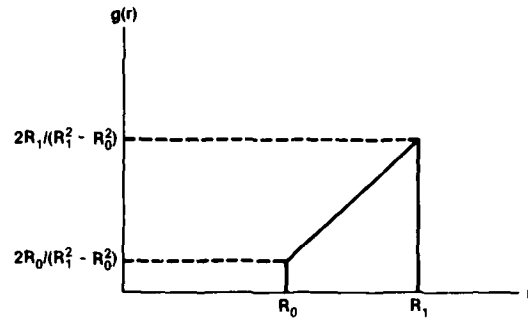


Figure 3. Radial density for interferer in a uniform deployment of interferers.

$$f(u) = \begin{cases} \alpha e^{-\alpha u} & , u > 0 \\ 0 & , u \leq 0 \end{cases} \quad (17)$$

where

$$\alpha = \frac{5}{4} \left( \frac{\beta}{D} \right)^4 \quad (18)$$

The deployment model described by equation (16) is not as easy to interpret as that described by equation (14). However, the model of equation (16) is not physically unreasonable, as seen from the plot in figure 4.

To simplify the expression for  $P_{jk}$ , we assume that  $P(\cdot)$  is a function of the sum of the interference powers entering the transmission channel and the sum of the interference powers entering the complementary channel. Mathematically, we can write

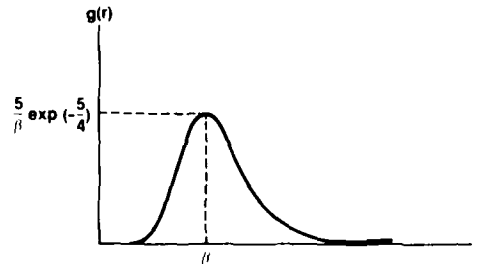


Figure 4. Radial density for interferer in a non-uniform deployment of interferers.

$$P(x_1, \dots, x_j, y_1, \dots, y_k) = P\left(\sum_{i=1}^j x_i, \sum_{i=1}^k y_i\right) \quad (19)$$

Substituting equations (17) and (19) into equation (4) yields

$$P_{jk} = \alpha^{j+k} \int_0^\infty \dots \int_0^\infty P\left(\sum_{i=1}^j x_i, \sum_{i=1}^k y_i\right) \times \exp\left[-\alpha\left(\sum_{i=1}^j x_i + \sum_{i=1}^k y_i\right)\right] \times dx_1 \dots dx_j dy_1 \dots dy_k \quad (20)$$

If  $j \geq 1$  and  $k \geq 1$ , we change the variables of integration to

$$u_\ell = \sum_{i=1}^\ell x_i, \quad \ell = 1, 2, \dots, j, \\ v_\ell = \sum_{i=1}^\ell y_i, \quad \ell = 1, 2, \dots, k \quad (21)$$

A straightforward calculation verifies that the Jacobian of the transformation is unity. Therefore,

$$P_{jk} = \alpha^{j+k} \int_0^\infty du_j \int_0^{u_j} du_{j-1} \dots \int_0^{u_2} du_1 \int_0^\infty dv_k \int_0^{v_k} dv_{k-1} \dots \int_0^{v_2} dv_1 P(u_j, v_k) \exp(-\alpha u_j - \alpha v_k), \quad j, k \geq 1 \quad (22)$$

Performing  $j + k - 2$  integrations and setting  $x = u_j$  and  $y = v_k$  for notational convenience, we get

$$P_{jk} = \frac{\alpha^{j+k}}{(j-1)!(k-1)!} \int_0^\infty \int_0^\infty P(x, y)$$

$$\times x^{j-1} y^{k-1} \exp(-\alpha x - \alpha y) dx dy, \quad j, k \geq 1 \quad (23a)$$

If  $j = 0$  or  $k = 0$ , one or both of the summations in equation (20) vanishes. By changing variables, we get

$$P_{0k} = \frac{\alpha^k}{(k-1)!} \int_0^\infty P(0, y) y^{k-1} \times \exp(-\alpha y) dy, \quad k \geq 1 \quad (23b)$$

$$P_{j0} = \frac{\alpha^j}{(j-1)!} \int_0^\infty P(x, 0) x^{j-1} \times \exp(-\alpha x) dx, \quad j \geq 1 \quad (23c)$$

$$P_{00} = P(0, 0) \quad (23d)$$

The function  $P(x, y)$ , which is the bit error probability for an FSK system, depends upon the nature of the interference and jamming. Once  $P(x, y)$  is determined, equations (3) and (23) completely determine  $P_b$ .

We obtain similar results for single-channel data modulations. Equation (17) can be used to express  $P_j$  as a single integral for all  $j$ . In equation (9), we assume that  $P(\cdot)$  is a function of the total interference power entering the transmission channel:

$$P(x_1, \dots, x_j) = P\left(\sum_{i=1}^j x_i\right) \quad (24)$$

By a derivation analogous to the derivation of equation (23), we obtain

$$P_j = \frac{\alpha^j}{(j-1)!} \int_0^\infty P(x) x^{j-1} \times \exp(-\alpha x) dx, \quad j \geq 1 \quad (25a)$$

$$P_0 = P(0) \quad (25b)$$

The function  $P(x)$  depends upon the nature of the interference and jamming. Once  $P(x)$  is determined, equations (8) and (25) completely determine  $P_b$ .

### 3. FREQUENCY-SHIFT KEYING

The bit error probability for a noncoherent FSK system operating in nonuniform Gaussian noise has been shown to be<sup>1</sup>

$$P_b(N_1, N_2) = \frac{N_2}{N_1 + N_2} \exp\left(-\frac{R_s}{N_1 + N_2}\right) \quad (26)$$

where  $R_s$  is the power of the desired signal,  $N_1$  is the total interference power in the transmission channel, and  $N_2$  is the total interference power in the complementary channel. If we assume that both the interference and jamming that enter a frequency-hopping receiver can be approximated by independent Gaussian processes with flat spectra over each affected channel, then  $P(x, y) = P_b[N_1(x), N_2(y)]$  is given by equation (26) with

$$\begin{aligned} N_1 &= N_t + N_{j1} + R_s x \\ N_2 &= N_t + N_{j2} + R_s y \end{aligned} \quad (27)$$

where  $N_t$  is the thermal noise power common to both channels,  $N_{j1}$  is the jamming power in the transmission channel,  $N_{j2}$  is the jamming power in the complementary channel,  $R_s x$  is the sum of the interference powers entering the transmission channel due to the hoppers, and  $R_s y$  is the sum of the interference powers entering the complementary channel due to the hoppers.

When jamming is absent, we set  $N_{j1} = N_{j2} = 0$  in equation (27) and substitute it into equation (26) to obtain  $P(x, y)$ . The result is

$$P(x, y) = \frac{N_t + R_s y}{2N_t + R_s(x + y)}$$

<sup>1</sup>D. J. Torrieri, *Frequency Hopping in a Jamming Environment*, U.S. Army DARCOT CM/CCM 78-2 (December 1978)

$$\times \exp\left[-\frac{R_s}{2N_t + R_s(x + y)}\right] \quad (28)$$

A repeater jammer is a device that intercepts a transmission, modulates and amplifies the waveform, and retransmits it at the same center frequency. We assume that the repeater jammer is close enough to the receiver and responds rapidly enough to interfere with the reception of the repeated bit.<sup>1</sup> In this case, the jamming power enters only the transmission channel, so that  $N_{j2} = 0$  and  $N_{j1} = N_j$ . Substituting equation (27) into equation (26) yields

$$\begin{aligned} P(x, y) &= \frac{N_t + R_s y}{2N_t + N_j + R_s(x + y)} \\ &\times \exp\left[-\frac{R_s}{2N_t + N_j + R_s(x + y)}\right] \quad (29) \end{aligned}$$

For partial-band jamming, the situation is more complicated since none, one, or both of the hopping channels may be jammed simultaneously.<sup>1</sup> Let  $D_0$  denote the event that neither of the two channels associated with a bit is jammed,  $D_1$  denote the event that one channel is jammed, and  $D_2$  denote the event that both channels are jammed. From elementary combinatorial considerations, the probability of event  $D_n$  is

$$\begin{aligned} P(D_n) &= \frac{\binom{2}{n} \binom{M-2}{J-n}}{\binom{M}{J}} \\ n &\leq J, J - n \leq M - 2, 0 \leq n \leq 2 \\ P(D_n) &= 0 \quad \text{otherwise} \quad (30) \end{aligned}$$

where  $J$  is the total number of jammed channels. We conclude that

$$P(x, y) = \sum_{n=0}^{n_1} \frac{\binom{2}{n} \binom{M-2}{J-n}}{\binom{M}{J}} S_n(x, y) \quad (31)$$

$$n_0 = \max(0, J + 2 - M), n_1 = \min(2, J),$$

where  $S_n(x, y)$  is the probability of bit error given  $x, y$ , and  $D_n$ . From equations (26) and (27), we obtain

$$S_0(x, y) = \frac{N_t + R_s y}{2N_t + R_s(x + y)} \times \exp\left[-\frac{R_s}{2N_t + R_s(x + y)}\right]. \quad (32)$$

Assuming that it is equally likely for either of the two channels to be jammed, we have

$$S_1(x, y) = \frac{1}{2} S_t(x, y) + \frac{1}{2} S_c(x, y), \quad (33)$$

where  $S_t(x, y)$  is the probability of bit error given  $x, y$ , and that the transmission channel alone is jammed; and  $S_c(x, y)$  is the probability of bit error given  $x, y$ , and that the complementary channel alone is jammed. Assuming that a jammed channel always receives jamming power  $N_j$ , equations (26) and (27) yield

$$S_t(x, y) = \frac{N_t + R_s y}{2N_t + N_j + R_s(x + y)} \times \exp\left[-\frac{R_s}{2N_t + N_j + R_s(x + y)}\right]. \quad (34)$$

$$S_c(x, y) = \frac{N_t + N_j + R_s y}{2N_t + N_j + R_s(x + y)} \times \exp\left[-\frac{R_s}{2N_t + N_j + R_s(x + y)}\right]. \quad (35)$$

Using equations (26) and (27) and setting  $N_{j1} = N_{j2} = N_j$  since both channels have the same jamming power, we obtain

$$S_2(x, y) = \frac{N_t + N_j + R_s y}{2N_t + 2N_j + R_s(x + y)}$$

$$\times \exp\left[-\frac{R_s}{2N_t + 2N_j + R_s(x + y)}\right]. \quad (36)$$

The substitution of equations (32) to (36) into equation (31) determines  $P(x, y)$  for partial-band jamming.

Because of the forms of the expressions for  $P(x, y)$  and  $S_n(x, y)$ , the double integral of equation (23a) can be reduced to a single integral, thereby greatly reducing the computational complexity of calculating  $P_b$ . When either equation (28) or equation (29) applies,  $P(x, y)$  has the form

$$P(x, y) = (a + by) h(x + y), \quad (37)$$

where  $h(\cdot)$  is a function of the sum of  $x$  and  $y$ , and  $a$  and  $b$  are independent of  $x$  and  $y$ . Similarly, for partial-band jamming, we have

$$P(x, y) = \sum_{n=n_0}^{n_1} \frac{\binom{2}{n} \binom{M-2}{J-n}}{\binom{M}{J}} (a_n + b_n y) \times h_n(x + y).$$

$$n_0 = \max(0, J + 2 - M), n_1 = \min(2, J). \quad (38)$$

We show how to simplify equation (23a) when equation (37) applies. The procedure when equation (38) applies is similar.

Substituting equation (37) into equation (23a) yields

$$P_{jk} = \frac{\alpha^{j+k}}{(j-1)!(k-1)!} \int_0^\infty \int_0^\infty (a + by) \times h(x + y) x^{j-1} y^{k-1} \exp(-\alpha x - \alpha y) \times dx dy, \quad j, k \geq 1. \quad (39)$$

We change the variables of integration to

$$u = y, \quad v = x + y. \quad (40)$$

The Jacobian of the transformation is unity. Thus, after regrouping, we have

$$P_{jk} = \frac{\alpha^{j+k}}{(j-1)!(k-1)!} \int_0^\infty h(v) e^{-\alpha v} \times F(v) dv, \quad j, k \geq 1, \quad (41)$$

where

$$F(v) = \int_0^v (a + bu)(v-u)^{j-1} u^{k-1} du. \quad (42)$$

The beta function is defined by

$$B(k, j) = \int_0^1 x^{k-1} (1-x)^{j-1} dx, \quad \text{Re } j > 0, \text{ Re } k > 0. \quad (43)$$

By changing the variable of integration to  $u = vx$ , we get

$$v^{j+k-1} B(k, j) = \int_0^v u^{k-1} (v-u)^{j-1} du. \quad (44)$$

Using this identity in equation (42) gives

$$F(v) = av^{j+k-1} B(k, j) + bv^{j+k} B(k+1, j). \quad (45)$$

Since  $j$  and  $k$  are positive integers, the beta function may be expressed as

$$B(k, j) = \frac{(j-1)!(k-1)!}{(j+k-1)!}. \quad (46)$$

Using equations (45) and (46) in equation (41) and simplifying, we obtain

$$P_{jk} = \frac{\alpha^{j+k}}{(j+k)!} \int_0^\infty h(v) e^{-\alpha v} v^{j+k-1} \times [a(j+k) + kbv] dv, \quad j+k \geq 1, \quad (47a)$$

$$P_{00} = P(0,0) \quad (47b)$$

We have derived equation (47a) for  $j, k \geq 1$ . However, combining equation (37) with equations (23b) and (23c), it follows that equation (47a) is valid for all  $j, k$  such that  $j+k \geq 1$ , as indicated.

All the above equations were derived assuming no coordination in the network. If the hoppers synchronize their choices of channels, then as many as  $M$  hoppers out of the  $N+2$  in the network can operate simultaneously without mutual interference. In general, if  $N+2 > M$ , then the probability of bit error assuming optimal network coordination is obtained by substituting  $N+2-M$  for  $N$  in the above equations.

Figures 5 to 15 show plots\* of the probability of bit error as a function of the number of interferers for various special cases. In figures 5 to 10, the deployment model of equation (14) is assumed. Using  $L=2$ , the calculated values of  $P_U$  and  $P_L$  — given by equations (4) to (7), (19), and appropriate expressions for  $P(x, y)$  — are usually so close that only one curve appears on the graph. Thus, this single curve can be considered a plot of  $P_b$ . The reason for the closeness is that  $dN/M$  is small in the cases considered. We assume that  $R_0/D = 0.2$  and  $R_1/D = 2.0$ . When jamming is absent or repeater jamming is present, it is convenient to use the parameter  $M_1 = M/d$ , which can be interpreted as the equivalent number of available channels.

Figures 5 and 6 illustrate the effects of mutual interference alone, assuming that jamming is absent. In figure 5,  $M_1 = 2000$  and the signal-to-thermal noise ratio,  $R_s/N_t$ , is a parameter. It is seen that if  $R_s/N_t \geq 15$  dB, the thermal noise level is irrelevant and the effect of the mutual interference predominates. Figure 6 shows the performance improvement that results when the number of channels is increased.

Figures 7 and 8 illustrate the effects of combined repeater jamming and mutual interference when  $R_s/N_t = 15$  dB and  $M_1 = 500, 1000$ , and

\*The program for the plots was written by H. R. Harrison, who was assisted by Benson J. King.



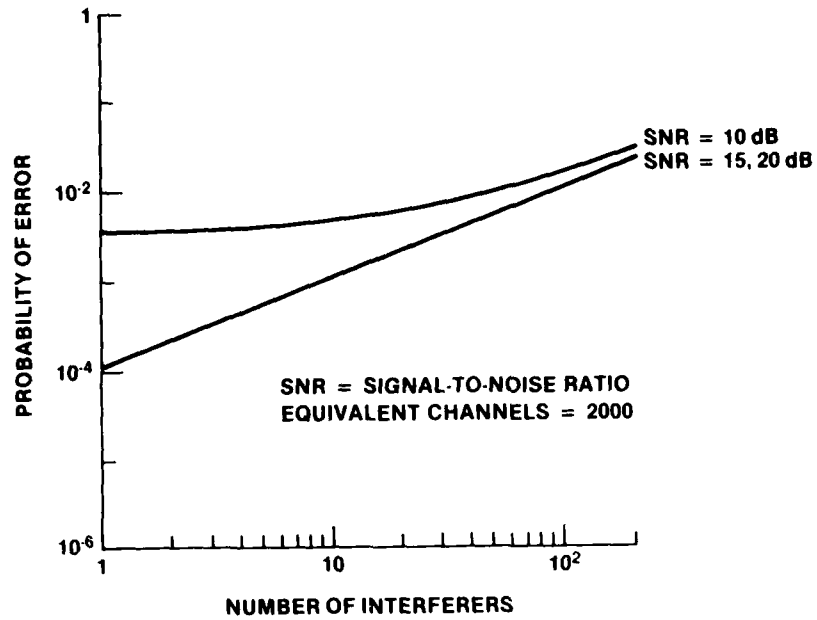


Figure 5. Bit error probability for uniform deployment, FSK, no jamming, and various signal-to-noise ratios.

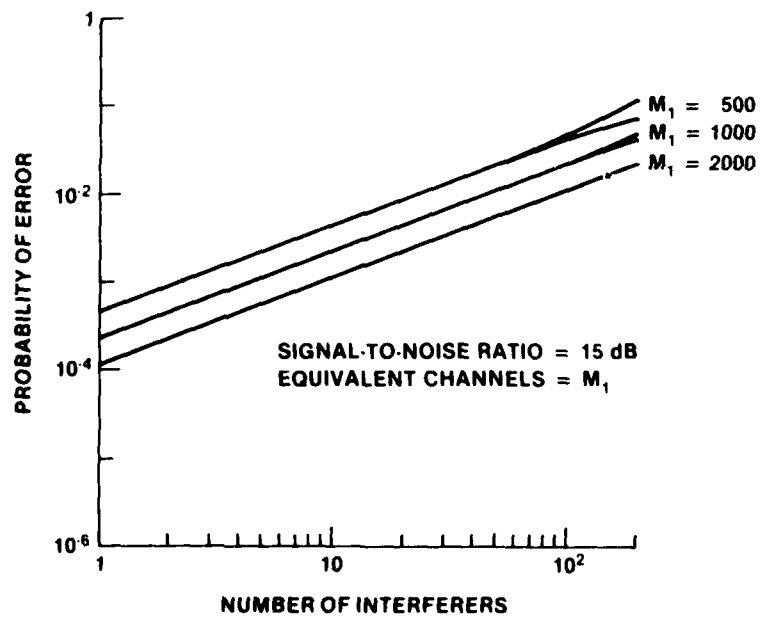


Figure 6. Bit error probability for uniform deployment, FSK, no jamming, and various numbers of equivalent channels.

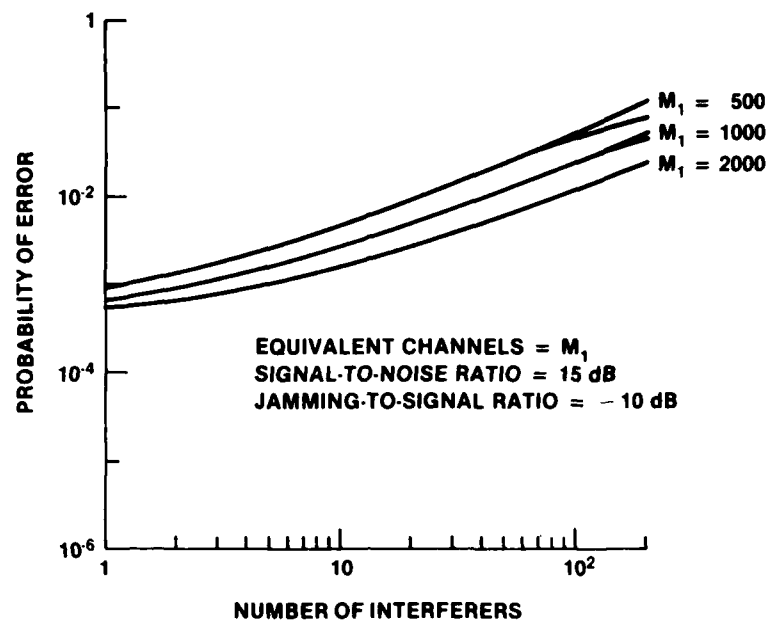


Figure 7. Bit error probability for uniform deployment, FSK, and weak repeater jamming.

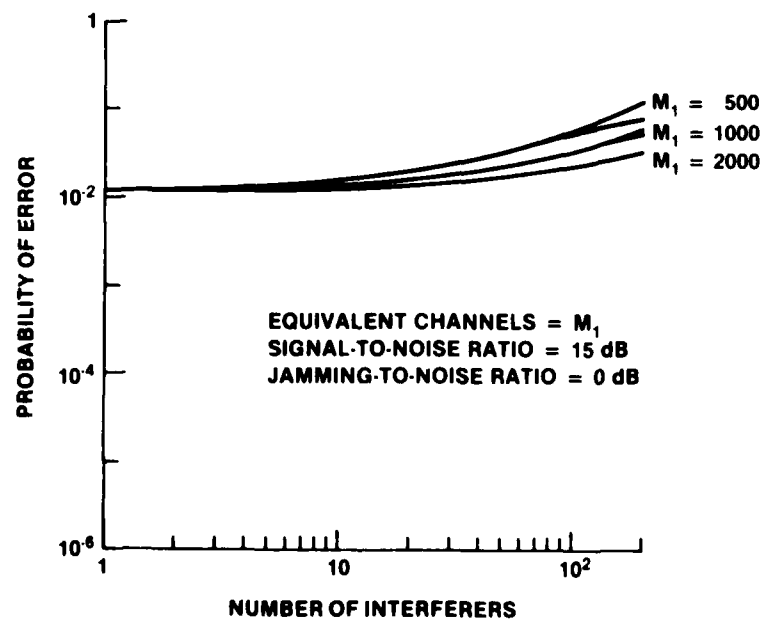


Figure 8. Bit error probability for uniform deployment, FSK, and strong repeater jamming.

2000. In figure 7, the jamming-to-signal ratio,  $N_j/R_s$ , is assumed to be  $-10$  dB. Although the addition of jamming raises the curves relative to figure 6, the effect of mutual interference is still pronounced. In figure 8, the jamming power is increased so that  $N_j/R_s = 0$  dB. In this case, the effect of the jamming is usually predominant.

Figures 9 and 10 illustrate the effects of combined partial-band jamming and mutual interference. The parameter  $\mu = J/M$  denotes the fraction of the available channels that contain jamming. For example, if  $\mu = 0.1$  and  $M = 2000$ , then 200 channels are jammed and 1800 do not contain jamming power. In figures 9 and 10, we assume  $R_s/N_t = 15$  dB,  $M = 2000$ ,  $d = 1$ , and  $\mu = 0.05, 0.1$ , and  $0.15$ . In figure 9, we assume  $N_j/R_s = -10$  dB. The effect of the mutual interference predominates. In figure 10, the jamming power is increased so that  $N_j/R_s = 0$  dB. In this case, the effect of  $\mu$  is significant over the range of  $N$ .

As the parameter  $dN/M$  increases, the values of  $P_U$  and  $P_L$  increasingly diverge. Thus, the deploy-

ment model of equation (16), although less intuitively appealing than the model of equation (14), becomes attractive since it provides a single value for  $P_b$ . In figures 11 to 15, the deployment model of equation (16) is assumed with  $\beta/D = 1.0$  and  $R_s/N_t = 15$  dB.

Figure 11 illustrates the effect of mutual interference in the absence of jamming with  $M_1 = 100, 300$ , and  $500$ . The effects of the addition of repeater jamming with  $N_j/R_s = -10$  dB and  $N_j/R_s = 0$  dB are shown in figures 12 and 13, respectively. In figures 14 and 15, the effects of combined partial-band jamming and mutual interference are illustrated. We assume that  $M = 500$ ,  $d = 1$ , and  $\mu = 0.05, 0.1$ , and  $0.15$ . In figure 14,  $N_j/R_s = -10$  dB; in figure 15,  $N_j/R_s = 0$  dB. The basic characteristics of figures 11 to 15 are similar to those of figures 6 to 10. In particular, the curves for  $M_1 = 500$  in figures 11 to 13 nearly coincide with the corresponding curves in figures 6 to 8. Thus, the details of the deployment model do not appear to be important in determining the bit error probability.

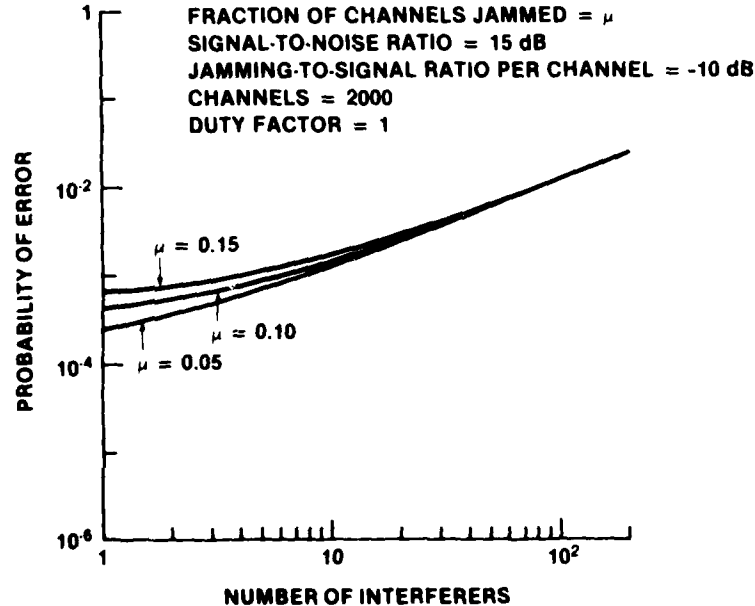


Figure 9. Bit error probability for uniform deployment, FSK, and weak partial-band jamming.

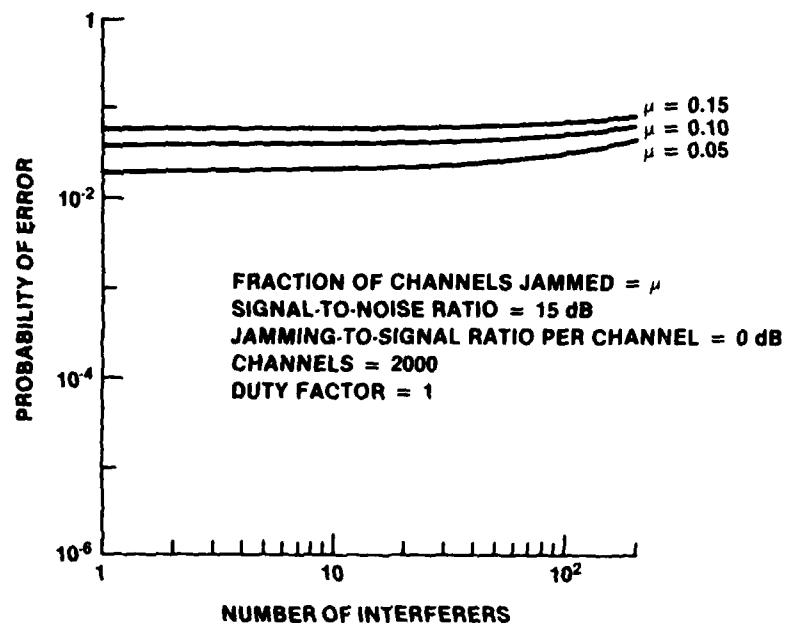


Figure 10. Bit error probability for uniform deployment, FSK, and strong partial-band jamming.

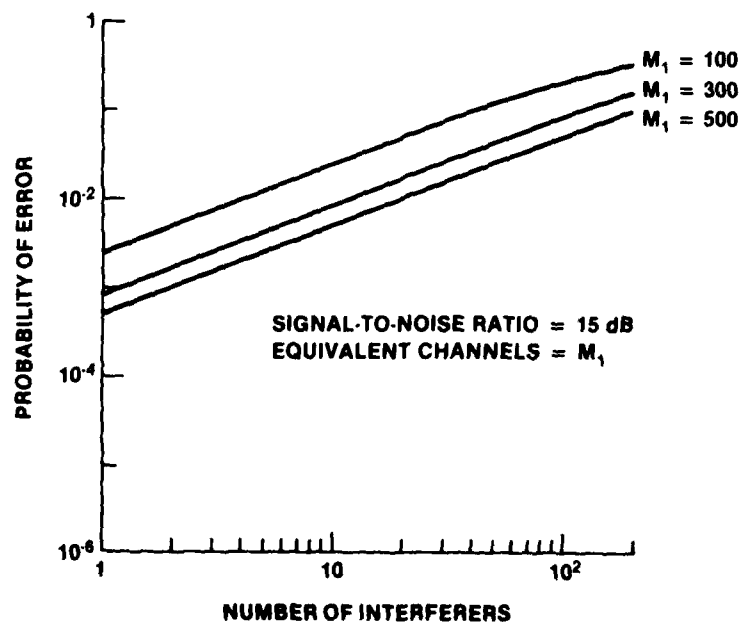


Figure 11. Bit error probability for nonuniform deployment, FSK, and no jamming.

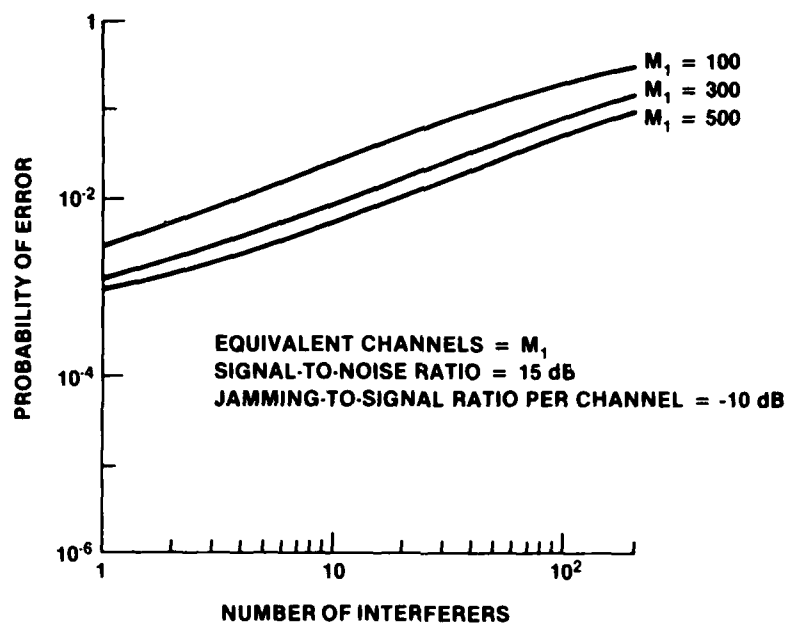


Figure 12. Bit error probability for nonuniform deployment, FSK, and weak repeater jamming.

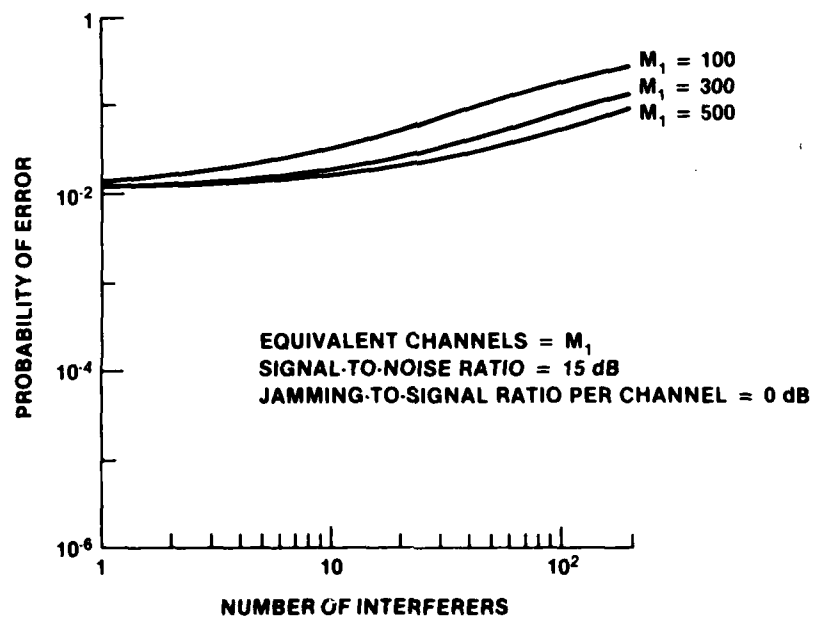


Figure 13. Bit error probability for nonuniform deployment, FSK, and strong repeater jamming.

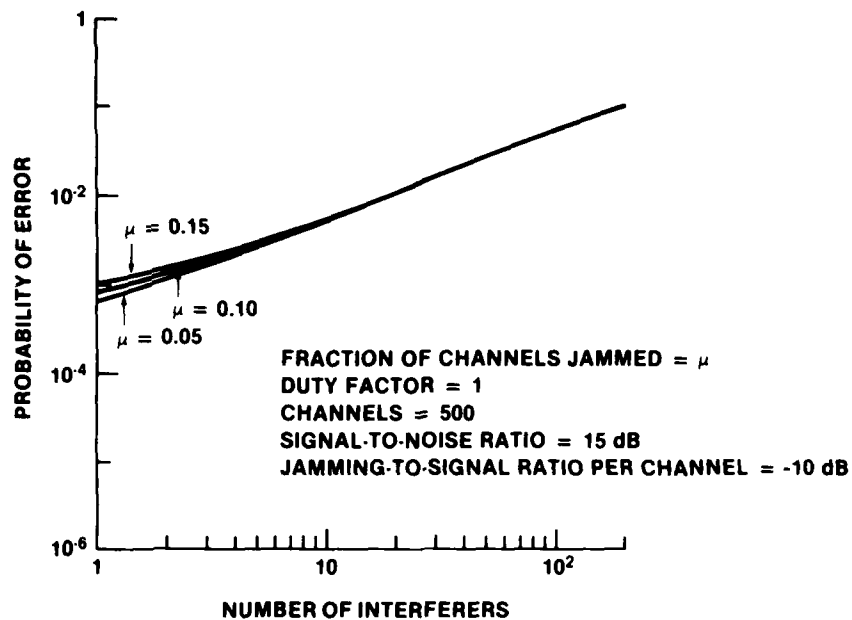


Figure 14. Bit error probability for nonuniform deployment, FSK, and weak partial-band jamming.

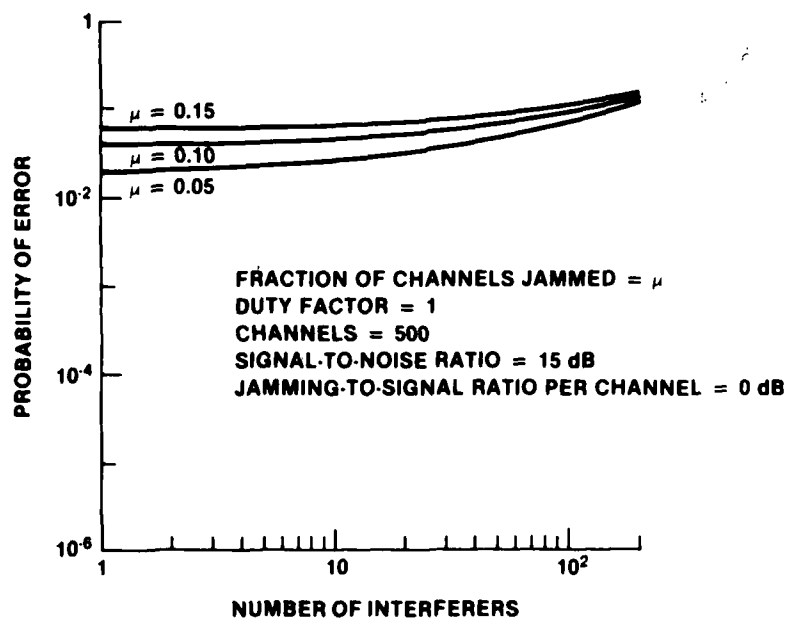


Figure 15. Bit error probability for nonuniform deployment, FSK, and strong partial-band jamming.

#### 4. MINIMUM-SHIFT KEYING

Signal phase coherence is difficult to maintain both from hop to hop in a transmitter and after dehoppping in a receiver. Consequently, unless the hopping rate is extremely low compared to the transmitted symbol rate, practical frequency-hopping systems require noncoherent or differentially coherent data modulations. In the latter case, the presence of a phase reference symbol during each hopping period, if necessary, causes a performance degradation relative to the ideal. However, if the hopping period includes enough symbols, this degradation becomes insignificant.

Continuous phase frequency-shift keying (CPFSK) is a data modulation that produces a transmitted signal with a compact spectrum and relatively low spectral splatter.<sup>3</sup> A special type of CPFSK of particular interest is minimum-shift keying (MSK). With MSK, both noncoherent reception and differentially coherent reception without an extra phase reference symbol are possible. Noncoherent reception with a discriminator<sup>4</sup> yields a bit error probability roughly approximated by equation (26) with  $N_1 = N_2$ . The probability of a bit error for differentially coherent reception is approximately given by<sup>5</sup>

$$P_b(N_1) = \frac{1}{2} \exp\left(-\frac{R_s}{N_1}\right) \quad (48)$$

where  $R_s$  is the power of the desired signal at the receiver, and  $N_1$  is the total interference power. If we assume that the data modulation is MSK with differentially coherent reception and that both the interference and jamming that enter a frequency-hopping receiver can be approximated by independent Gaussian processes with flat spectra over each affected channel, then  $P(x) = P_b[N_1(x)]$  is given by equation (48) with

$$N_1 = N_t + N_j + R_s x \quad (49)$$

where  $N_t$  is the thermal noise power,  $N_j$  is the jamming power, and  $R_s x$  is the sum of the interference powers due to the other network hoppers.

When jamming is absent, we set  $N_j = 0$  in equation (49) and substitute it into equation (48) with the result that

$$P(x) = \frac{1}{2} \exp\left(-\frac{R_s}{N_t + R_s x}\right) \quad (50)$$

For repeater jamming, we obtain

$$P(x) = \frac{1}{2} \exp\left(-\frac{R_s}{N_t + N_j + R_s x}\right) \quad (51)$$

where  $N_j$  is the jamming power that passes the receiver bandpass filter. The same formula holds for barrage jamming of the total bandwidth over which frequency hopping occurs.

For partial-band jamming, let  $D_0$  denote the event that the transmission channel is not jammed, and  $D_1$  denote the event that it is jammed. If  $J$  of the possible channels are jammed, the probabilities of these events are

$$\begin{aligned} P(D_0) &= 1 - \frac{J}{M} \\ P(D_1) &= \frac{J}{M} \end{aligned} \quad (52)$$

It follows that

$$P(x) = \left(1 - \frac{J}{M}\right) S_0(x) + \frac{J}{M} S_1(x) \quad (53)$$

where  $S_n(x)$  is the probability of bit error given  $x$  and  $D_n$ . From the definition of  $S_0(x)$  and equation (51), we obtain

$$S_0(x) = \frac{1}{2} \exp\left(-\frac{R_s}{N_t + R_s x}\right) \quad (54)$$

Assuming that a jammed channel always receives jamming power  $N_j$ , equations (48) and (49) imply

<sup>3</sup>S. Haskin, *Communication Systems*, Wiley (1978).

<sup>4</sup>R. Gagliardi, *Introduction to Communication Engineering*, Wiley (1978).

<sup>5</sup>T. Masumura et al., Differential Detection of MSK with Nonredundant Error Correction, *IEEE Trans. Comm.* COM-27 (June 1979), 912.

$$S_1(x) = \frac{1}{2} \exp\left(-\frac{R_s}{N_t + N_j + R_s x}\right) \quad (55)$$

Equations (53) to (55) determine  $P(x)$  for partial-band jamming.

Figures 16 to 26 show plots of the probability of bit error as a function of the number of interferers for various special cases. In figures 16 to 21, the deployment model of equation (14) is assumed. Using  $L = 2$ , equations (9) to (12), (24), and appropriate expressions for  $P(x)$ , we obtain the values of  $P_U$  and  $P_L$ , which are usually so close that one curve appears on the graph. This curve can be interpreted as a plot of  $P_b$ . To expedite comparison with the corresponding results for noncoherent FSK, we choose  $R_0/D = 0.2$  and  $R_1/D = 2.0$ .

Figures 16 and 17 illustrate the effects of mutual interference alone, assuming that jamming is absent. From figure 16, it is seen that the thermal noise level does not significantly affect  $P_b$  unless  $R_s/N_t$  is less than approximately 10 dB. Figure 17 is similar to figure 6 except that  $P_b$  is lower for MSK than for FSK when other parameter values are identical. The difference between the corresponding curves is small when  $R_s/N_t = 15$  dB, but is substantial when  $R_s/N_t = 10$  dB.

Figures 18 and 19 illustrate the effects of repeater jamming and mutual interference when  $R_s/N_t = 15$  dB and  $M_t = 500, 1000$ , and 2000. When the jamming is weak, as in figure 18,  $P_b$  is similar in the MSK and FSK cases. However, when the jamming is strong, the impact on systems with MSK is far greater than on corresponding systems with FSK, as a comparison of figures 19 and 8 shows.

Figures 20 and 21 illustrate the effects of simultaneous partial-band jamming and mutual interference with  $R_s/N_t = 15$  dB,  $M = 2000$ ,  $d = 1$ , and  $\mu = 0.05, 0.1$ , and 0.15. The curves are qualitatively similar to those in figures 9 and 10 except that  $P_b$  is lower for MSK.

So that a single curve may be exhibited for  $P_b$  when  $dN/M$  is relatively large, the deployment model of equation (16) is used in figures 22 to 26. We assume that  $\beta/D = 1.0$  and  $R_s/N_t = 15$  dB. The basic characteristics of figures 22 to 26 are similar to those of figures 17 to 21.

The comparison of figures 16 to 26 with figures 5 to 15 leads to the conclusion that systems with MSK potentially perform better than comparable systems with FSK except when strong repeater jamming is present. In that case, systems with MSK may be disrupted, while corresponding systems with FSK operate acceptably.



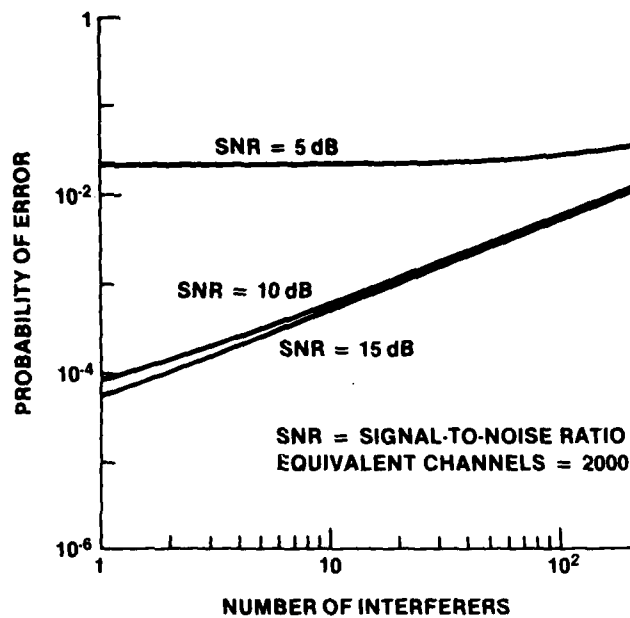


Figure 16. Bit error probability for uniform deployment, MSK, no jamming, and various signal-to-noise ratios.

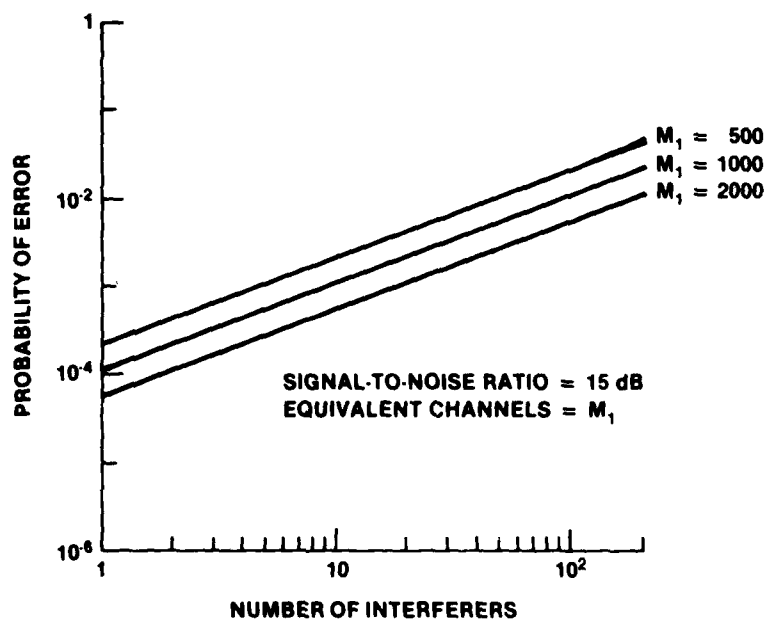


Figure 17. Bit error probability for uniform deployment, MSK, no jamming, and various numbers of equivalent channels.

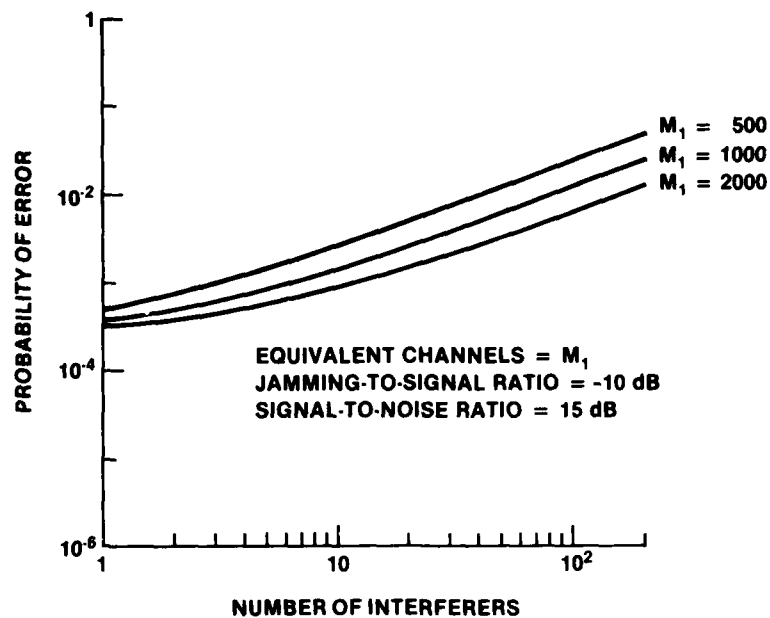


Figure 18. Bit error probability for uniform deployment, MSK, and weak repeater jamming.

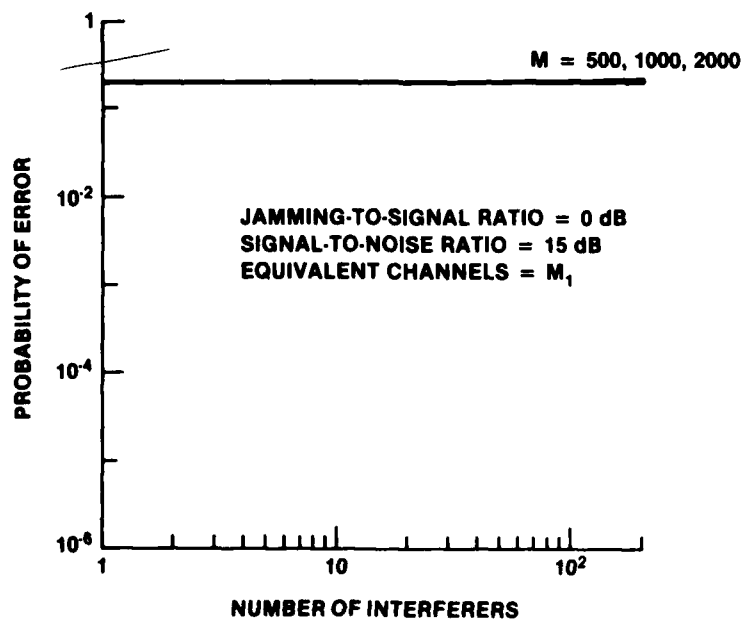


Figure 19. Bit error probability for uniform deployment, MSK, and strong repeater jamming.

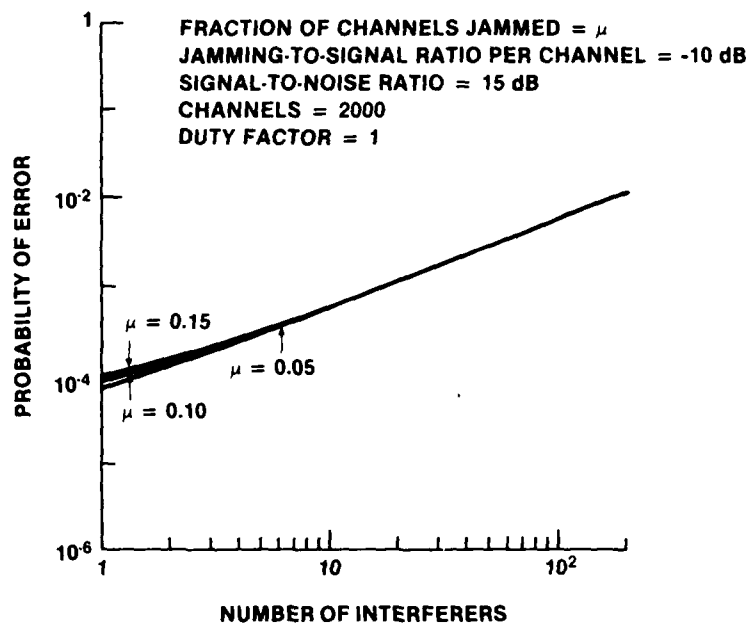


Figure 20. Bit error probability for uniform deployment, MSK, and weak partial-band jamming.

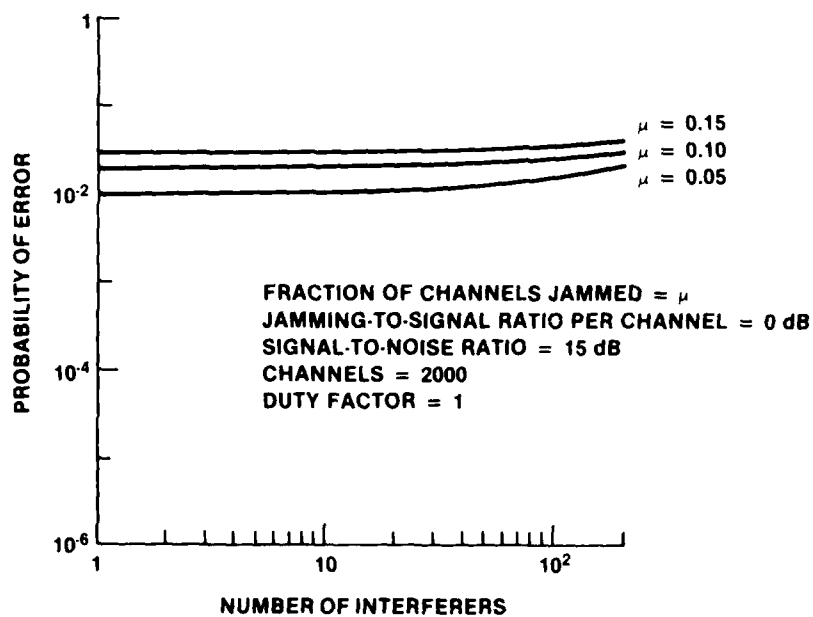


Figure 21. Bit error probability for uniform deployment, MSK, and strong partial-band jamming.

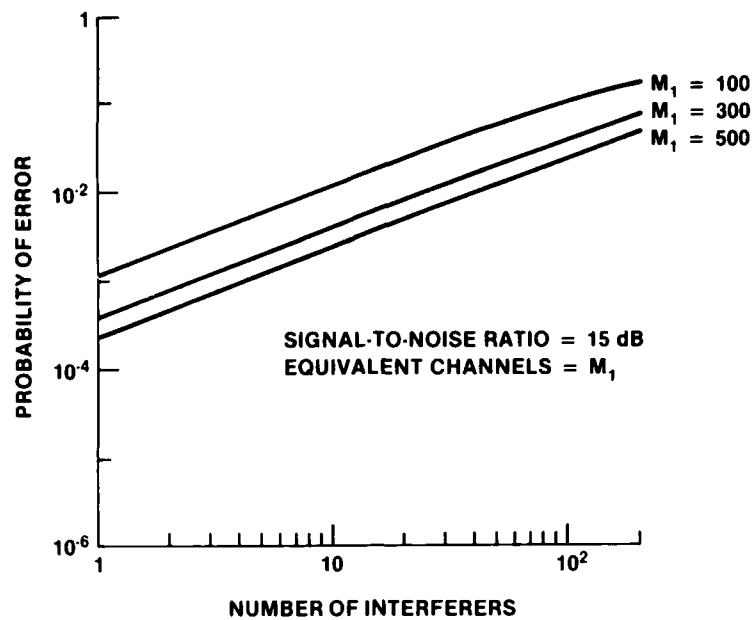


Figure 22. Bit error probability for nonuniform deployment, MSK, and no jamming.

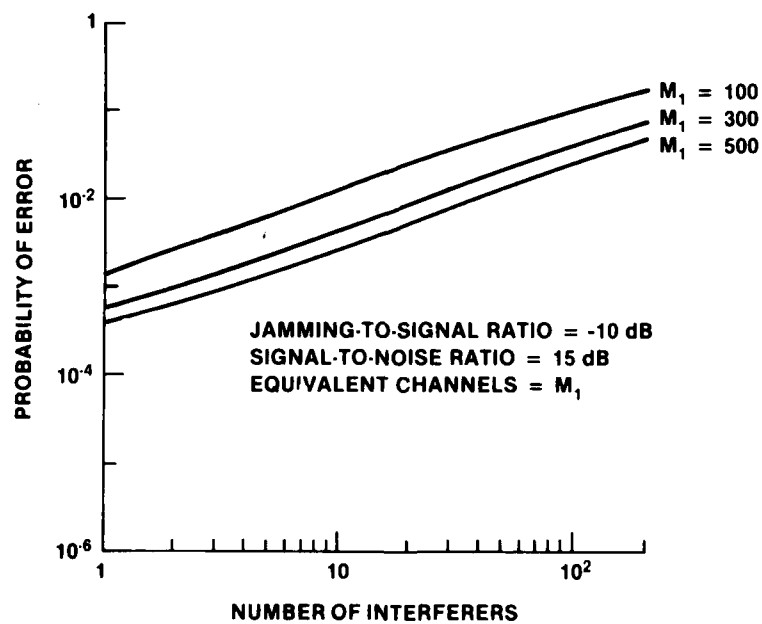


Figure 23. Bit error probability for nonuniform deployment, MSK, and weak repeater jamming.

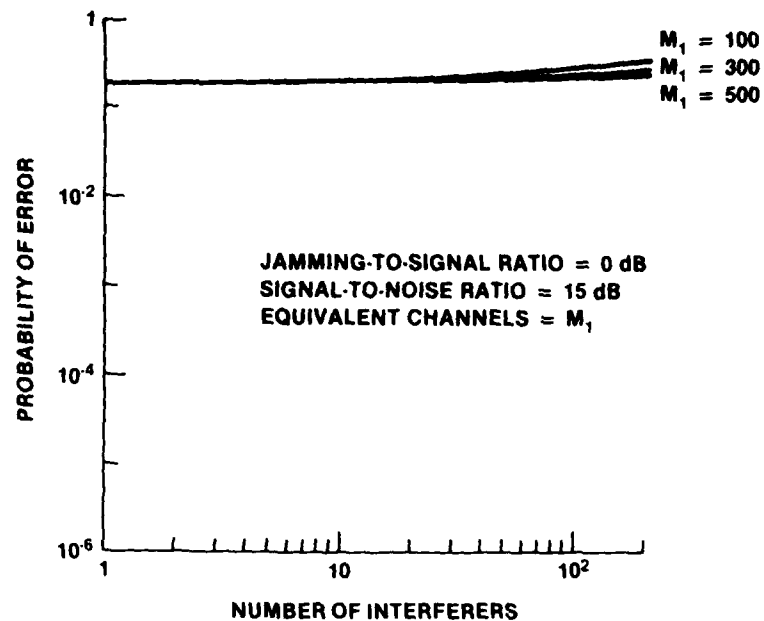


Figure 24. Bit error probability for nonuniform deployment, MSK, and strong repeater jamming.

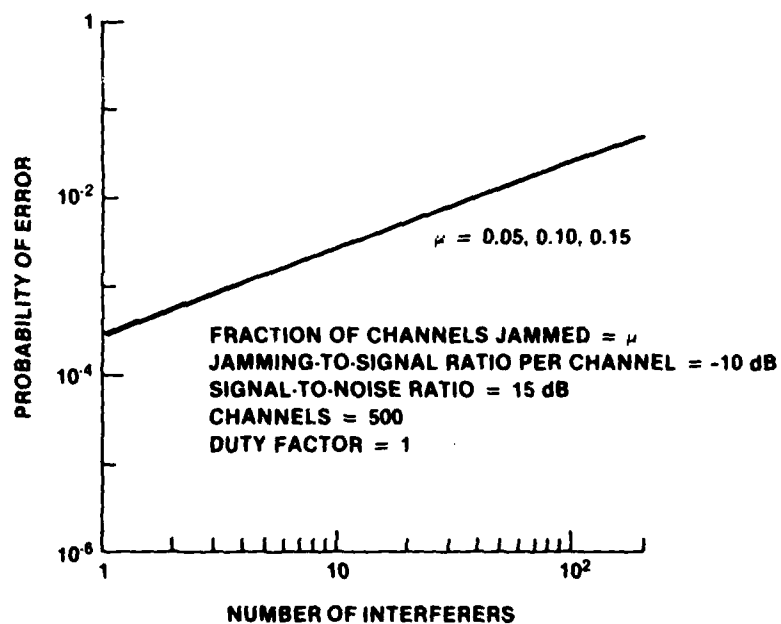


Figure 25. Bit error probability for nonuniform deployment, MSK, and weak partial-band jamming.

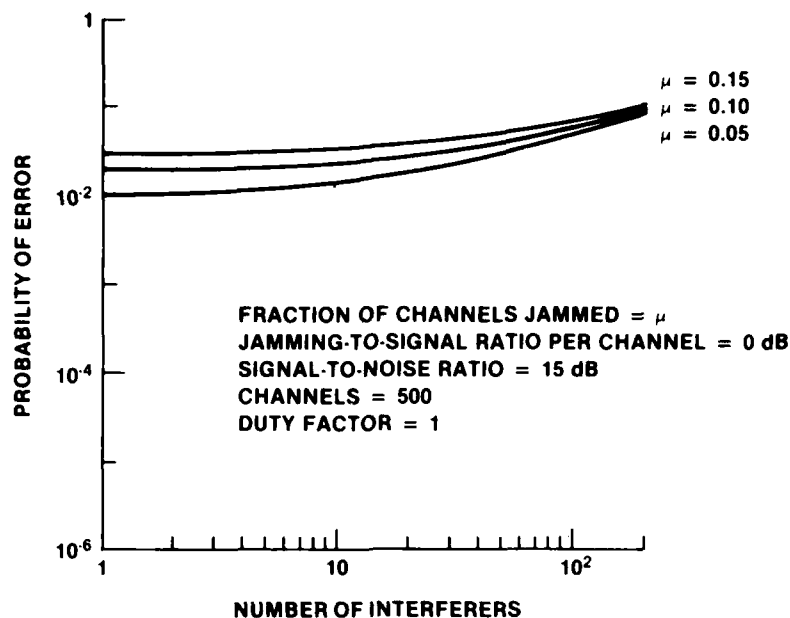


Figure 26. Bit error probability for nonuniform deployment, MSK, and strong partial-band jamming.

## 5. SPECTRAL SPLATTER

Spectral splatter is the spectral overlap in extraneous channels produced by a time-limited transmitted pulse. Whether or not spectral splatter is significant in causing bit errors in a network depends upon the deployment, the hopping rate, the frequency separation between channels, and the spectrum of the transmitted signals.

If the frequency-hopping systems hop with each transmitted symbol, then the hopping rate strongly influences the transmitted spectrum and the number of available channels. If the hopping rate is slower than the transmitted symbol rate, then the hopping rate influences the spectrum indirectly through the switching time, which is defined to be the part of the hopping period during which the frequency synthesizer is not operating, plus any rise time or fall time not directly due to the data modulation. The nonzero switching time decreases the transmitted symbol period, which in turn affects the transmitted spectrum.

If the total bandwidth over which hopping occurs is fixed, increasing the frequency separation between channels reduces the number of available channels. As a result, the hopping systems become more vulnerable to mutual interference and certain types of jamming. Thus, pulse shaping and the appropriate choice of data modulation are often important in limiting spectral splatter.

If FSK is the data modulation, the transmission of approximately Gaussian or raised cosine pulses can greatly reduce the spectral splatter. However, once a Gaussian pulse is generated, it must pass through a power amplifier before transmission. Since an amplifier must often operate in its nonlinear region for efficiency, some clipping of the pulse results. The clipping can considerably increase the splatter so that the net benefit from the pulse shaping is significantly reduced.

If an approximately constant envelope signal is generated, the spectral effects of the power amp

lifier are usually negligible. Since a constant envelope signal that has a compact spectrum is produced by MSK, this data modulation is an attractive choice in frequency-hopping networks with a potential spectral splatter problem.

Spectral splatter emanating from a transmission channel may significantly affect not only the two adjacent channels, but also other channels farther in frequency from the transmission channel. To reduce splatter in the latter channels, variations of MSK are possible. The MSK format can be generalized by appropriate shaping of the data bits in such a way that the constant envelope, bit error probability, and other desirable features of MSK are largely retained. The class of generalized MSK signals include signals with much faster spectral roll-offs than conventional MSK signals.<sup>6,9</sup>

Assuming generalized MSK data modulation, we derive equations for the bit error probability when the splatter is significant only in the two channels adjacent to the transmission channel. The generalization of the derivation to the case in which many channels are affected is straightforward but notationally complicated; the resulting equations are expensive to evaluate with a computer. Thus, a rough approximation of the bit error probability for multiple-channel splatter is given subsequently.

The derivation that follows parallels the derivation of equations (1) to (7). Consider the transmission of a bit. The event  $B_{jk}$  is the event that  $j$  of the interferers use the transmission channel and each of  $k$  interferers uses one of the two channels adjacent to the transmission channel. We denote the probability of  $B_{jk}$  by  $P_s(B_{jk})$ . The probability of a bit error given  $B_{jk}$  is denoted by  $P_s(j, k)$ . Thus, the probability of a bit error is

$$P_b = \sum_{j=0}^N \sum_{k=0}^{N-j} P(B_{jk}) P_s(j, k) \quad (56)$$

The probability that power from an interferer enters the transmission channel is  $d/M$ . We assume that  $M$  is sufficiently large that we may neglect the fact that a channel at one of the ends of the total bandwidth has only one adjacent channel instead of two. Consequently, the probability that the power from an interferer enters one of the two adjacent channels is  $2d/M$ . The probability that the power enters neither the transmission channel nor the adjacent channels is  $1 - 3d/M$ . There are

$$\binom{N}{j} \binom{N-j}{k}$$

ways to select one set of  $j$  interferers and another set of  $k$  interferers when  $j + k \leq N$ . Thus,

$$P(B_{jk}) = \left(\frac{d}{M}\right)^j \left(\frac{2d}{M}\right)^k \left(1 - \frac{3d}{M}\right)^{N-j-k} \times \binom{N}{j} \binom{N-j}{k} \quad (57)$$

Substituting into equation (56) yields

$$P_b = \sum_{j=0}^N \sum_{k=0}^{N-j} 2^k \left(1 - \frac{3d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} P_s(j, k) \quad (58)$$

From this equation, we obtain lower and upper bounds. Since  $0 \leq P_s(j, k) \leq 1/2$ ,

$$P_L = \sum_{j=0}^N \sum_{k=0}^{N-j} 2^k \left(1 - \frac{3d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} P_s(j, k) \quad (59)$$

<sup>6</sup>M. K. Simon, A Generalization of Minimum Shift Keying (MSK) Type Signaling Based Upon Input Data Symbol Pulse Shaping, *IEEE Trans. Comm. COM-24* (August 1976), 845.

<sup>7</sup>M. Razzel and S. Pasupathy, Spectral Shaping in Minimum Shift Keying (MSK) Type Signals, *IEEE Trans. Comm. COM-26* (January 1978), 189.

<sup>8</sup>B. Bazin, A Class of MSK Baseband Pulse Formats with Sharp Spectral Roll Off, *IEEE Trans. Comm. COM-27* (May 1979), 826.

<sup>9</sup>J. Amoroso, The Use of Quasi-Bandlimited Pulses in MSK Transmission, *IEEE Trans. Comm. COM-27* (October 1979), 1616.

$$P_U = P_L + \frac{1}{2} P(j + k > L) \quad (60)$$

$$P(j + k > L) = \sum_{j=0}^N \sum_{\substack{k=0 \\ j+k > L}}^{N-j} 2^k \left(1 - \frac{3d}{M}\right)^{N-j-k} \times \left(\frac{d}{M}\right)^{j+k} \binom{N}{j} \binom{N-j}{k} \quad (61)$$

If interferer  $i$  is using the transmission channel, the ratio of the power from interferer  $i$  to the power of the desired signal at the receiver is denoted by  $x_i$ . If the interferer  $i$  is using an adjacent channel, the ratio of the power from interferer  $i$  to the power of the desired signal at the receiver is denoted by  $z_i$ . Let  $P(x_1, \dots, x_j, z_1, \dots, z_k)$  denote the probability of a bit error given that  $x_1, x_2, \dots, x_j, z_1, z_2, \dots, z_k$  are the interference-to-signal ratios caused by  $j$  interferers using the transmission channel and  $k$  interferers using an adjacent channel. If the interfering signals are modeled as independent Gaussian processes,

$$P(x_1, \dots, x_j, z_1, \dots, z_k) = P\left(\sum_{i=1}^j x_i + \sum_{i=1}^k z_i\right) \quad (62)$$

Let  $f(u)$  denote the probability density function for an interference-to-signal ratio given that the interference enters the transmission channel. Let  $f_1(u)$  denote the probability density function for an interference-to-signal ratio given that the interference enters an adjacent channel. We denote by  $K$  the ratio of the power due to an adjacent-channel interferer to the corresponding power that would exist if the interferer were using the transmission channel.

Note that  $K$  is a constant independent of the index  $i$  because each hopper in the network is assumed to produce an identical spectrum. Thus,  $z_i = Kx'_i$ , where  $x'_i$  has the density  $f(u)$ . Since  $f_1(u)$  is the density for  $z_i$ , elementary probability theory gives

$$f_1(u) = \frac{1}{K} f\left(\frac{u}{K}\right) \quad (63)$$

Since each interferer is located and hops independently of the other interferers, equations (62) and (63) and the definition of  $P_s(j, k)$  imply that

$$P_s(j, k) = \left(\frac{1}{K}\right)^k \int_0^\infty \dots \int_0^\infty P_s\left(\sum_{i=1}^j x_i + \sum_{i=1}^k z_i\right) \prod_{i=1}^j f(x_i) \prod_{i=1}^k f\left(\frac{z_i}{K}\right) dx_1 \dots dx_j dz_1 \dots dz_k \quad (64)$$

An alternative form of this equation results if we change variables to  $y_i = z_i/K$ . We get

$$P_s(j, k) = \int_0^\infty \dots \int_0^\infty P\left(\sum_{i=1}^j x_i + K \sum_{i=1}^k y_i\right) \prod_{i=1}^j f(x_i) \prod_{i=1}^k f(y_i) dx_1 \dots dx_j dy_1 \dots dy_k \quad (65)$$

In most practical deployments, only a few interferers will be close enough to a receiver to cause significant splatter when hopping in channels beyond the adjacent channels. Suppose there are  $\Gamma$  of these close interferers and also  $N$  other interferers uniformly deployed beyond a minimum radius  $R_0$  such that splatter is insignificant when the interferers hop in channels beyond the adjacent channels (there are  $N + 2 + \Gamma$  hoppers in the network). To make rough estimates of the bit error probability, we estimate the number



of nearby channels that can be significantly affected by splatter from the closest interferer and denote this even number by  $2q$ . We assume that if one or more of the interferers hops in the transmission channel or the  $2q$  channels closest to it, then a bit error probability of  $1/2$  is produced. If no close interferer hops into these  $2q + 1$  channels, then the bit error probability is determined by the interferers beyond  $R_0$ . We ignore the effects of a close interferer and an interferer beyond  $R_0$  simultaneously hopping into the transmission channel or the adjacent channels. Thus, the bit error probability,  $P'_b$ , is roughly approximated by

$$P'_b = P_b + \frac{1}{2} \left( 1 - \left[ 1 - \frac{(2q+1)d}{M} \right]^\Gamma \right) \quad (66)$$

where  $P_b$  is the bit error probability assuming a uniform deployment beyond a minimum radius and splatter from adjacent channels only. Alternatively, if the exact deployment of the  $\Gamma$  close interferers is known,  $P'_b$  can be approximated by  $P_b$  plus the sum of the bit error probabilities that would be induced by each interferer alone. This approximation is reasonable if  $(2q+1)d/M \ll 1$ .

We give an example of the effect of adjacent-channel spectral splatter when the parameter  $K$  (called the adjacent splatter ratio) equals 0.05. This value of  $K$  might arise in the following way. If the channels are designed to capture 90 percent of the intended signal's power, then less than five percent or 0.05 of the power can fall into one of the adjacent channels. If the data modulation is conventional MSK, the channel bandwidth required is approximately  $0.8/T_b$ , where  $T_b$  is the duration of a transmitted bit, allowing for the switching time. With this bandwidth value, the effect of splatter on a transmitted bit from channels farther in frequency than the adjacent channels is usually negligible if  $R_0/D = 0.2$  in a uniform deployment.

Figure 27 shows the bit error probability as a function of the number of interferers for  $K = 0.05$ ,  $L = 2$ ,  $R_0/D = 0.02$ ,  $R_1/D = 2.0$ , and no jamming. The values of  $P_U$  and  $P_L$  sometimes diverge. The result of the splatter is to raise the curves by a small amount relative to the corres-

ponding curves with no splatter, which are depicted in figure 17. When jamming is present, similar small changes due to splatter occur.

As an example of the use of equation (66), we let  $\Gamma = 0, 1, 3$ , and 5. To determine  $q$ , we must examine the average spectrum caused by MSK pulses. Suppose the interferers can be located as close as  $0.01 D$ . Then equation (13) implies a maximum interference-to-signal ratio, called the near-far ratio, equal to 80 dB. Such a large near-far ratio causes significant splatter in many channels if conventional MSK is used. Thus, we assume that sinusoidal frequency-shift keying (SFSK) is the type of generalized MSK used. If the channel bandwidth is  $1.2/T_b$ , then spectral plots indicate that  $q = 4$  is appropriate and  $K \approx 0.02$ . Figure 28 shows the resulting bit error probability for  $L = 2$ ,  $R_0/D = 0.2$ ,  $R_1/D = 2.0$ ,  $M_1 = 2000$ , and no jamming. The impact of the close interferers is predominant unless  $N$  is large.

The calculation of  $K$  and  $q$  is facilitated if we use plots of the fractional out-of-band power, defined as

$$F(B) = 1 - \frac{\int_B^B A(f) df}{\int_{-\infty}^{\infty} A(f) df}, \quad B \geq 0 \quad (67)$$

where  $A(f)$  is the power spectral density of the equivalent low-pass generalized MSK waveform and  $B$  is the bandwidth. The available plots<sup>6,8</sup> depict  $F(B)$  in decibels as a function of  $B$  in units of  $1/T_b$ . The fractional power within a transmission channel of bandwidth  $W$  is given by

$$K_i = 1 - F\left(\frac{W}{2}\right) \quad (68)$$

Let the index  $i$  denote a channel that is  $i$  channels removed from the transmission channel. If the

<sup>6</sup>M. K. Simon, "A Generalization of Minimum Shift Keying (MSK): Type Signaling Based Upon Input Data Symbol Pulse Shaping," *IEEE Trans. Comm. COM-24* (August 1976), 845.

<sup>7</sup>M. Rubel and S. Pasupathy, "Spectral Shaping in Minimum Shift Keying (MSK): Type Signals," *IEEE Trans. Comm. COM-26* (January 1978), 189.

<sup>8</sup>R. Buzon, "A Class of MSK Baseband Pulse Formats with Sharp Spectral Roll Off," *IEEE Trans. Comm. COM-27* (May 1979), 826.

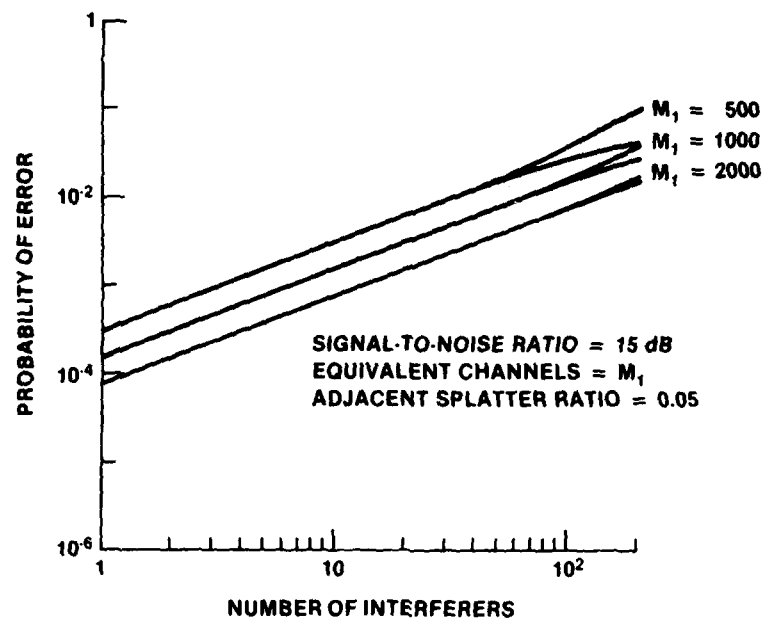


Figure 27. Bit error probability for uniform deployment, MSK, no jamming, and adjacent-channel splatter.

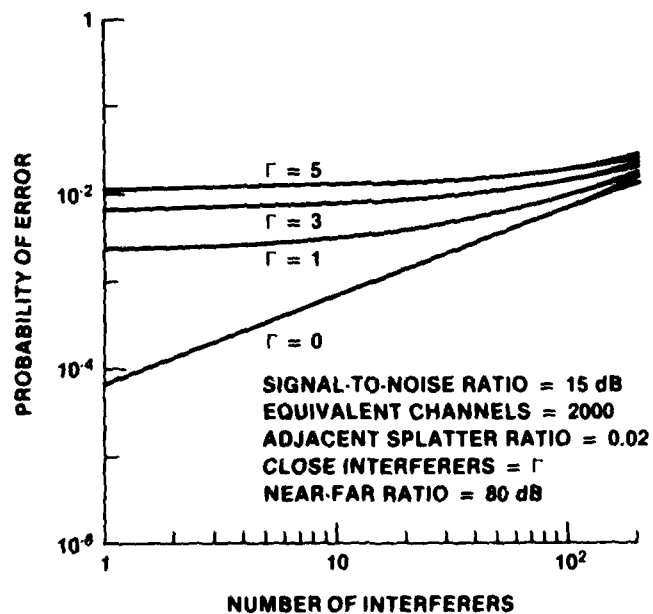


Figure 28. Bit error probability for uniform deployment, SFSK, no jamming, and close interferers.

channel separation is  $W$ , the fractional power intercepted by channel  $i$  due to spectral splatter from the transmission channel is given by

$$K_i = \frac{1}{2} \left[ F\left(iW - \frac{W}{2}\right) - F\left(iW + \frac{W}{2}\right) \right], \quad i = 1, 2, \dots \quad (69)$$

The factor  $1/2$  is due to the fact that there are two channels, one on each side, that are  $i$  channels removed from the transmission channel. From equations (68) and (69), we obtain the adjacent splatter ratio,

$$K = \frac{K_i}{K_0} = \frac{F\left(\frac{W}{2}\right) - F\left(\frac{3W}{2}\right)}{2\left[1 - F\left(\frac{W}{2}\right)\right]} \quad (70)$$

The parameter  $q$  can be defined as the largest index  $i$  for which  $K_0/K_i$  is less than the near-far ratio.

## 6. WORD ERROR PROBABILITY

The word error probability is usually a more useful measure of communication system performance than the bit error probability. Derivations of word error probabilities for FSK in the absence of mutual interference are given elsewhere.<sup>1</sup> Here, we use similar methods to derive the word error probability of a frequency-hopping system with single-channel, binary data modulation in the presence of mutual interference. We give specific examples for MSK data modulation.

When a block code is used for error correction, each uncoded word of  $w$  bits is represented by a code word of  $c$  bits, where  $c > w$ . Depending upon the code,  $r$  or more received bits of a code word must be in error for a word error to occur at the decoder output. If the word duration is preserved after encoding, the duration of a transmitted encoded bit is reduced and the channel bandwidths must be increased. Thus, if the total bandwidth is

not changed, the number of available channels for frequency hopping,  $M$ , is reduced relative to the number of channels,  $M_u$ , that would be available in the absence of coding. The thermal noise power,  $N_t$ , is increased relative to the thermal noise power,  $N_{tu}$ , that would be present in the absence of coding. We have

$$M = \text{int} \left( \frac{M_u w}{c} \right), \quad (71)$$

$$N_t = \frac{N_{tu} c}{w},$$

where  $\text{int}(x)$  is the largest integer contained in  $x$ . The coding is effective when its error-correction capability is sufficient to overcome the degradation implied by these equations.

To analyze the effects of jamming, we distinguish between slow and fast frequency hopping. Fast frequency hopping occurs if there is a frequency hop for each transmitted symbol. Thus, for binary communications the hopping rate equals or exceeds the data (message) bit rate. Slow frequency hopping occurs if two or more symbols are transmitted in the time interval between frequency hops.

When some, but not all, of the channels are jammed, the word error rates for block-encoded, binary, slow frequency-hopping systems are usually higher than for corresponding fast frequency-hopping systems. The reason is that the communicators hop out of a jammed channel after each transmitted bit in fast systems, whereas the communicators dwell in a jammed channel for several bits before hopping in a slow system. Consequently, the errors in slow systems tend to occur in bursts that may overwhelm the error-correcting capability of the block code. One remedy is to interleave the encoded bits before transmission so that each bit of a word is associated with a different frequency. After deinterleaving, the error-correcting capability of the block code equals that of the same block code used in a fast system. Thus, by employing additional hardware slow systems can give the same word error

<sup>1</sup>D. J. Torrieri, *Frequencies Hopping in a Jamming Environment*, U.S. Army DARCOT CM CCM 78 2 (December 1978).

rates as fast systems in the presence of partial-band jamming. Bit interleaving in fast systems permits the correction of bursts of errors due to high-power pulsed jamming over the total bandwidth.

Most single-channel data modulations are not very practical for fast frequency-hopping systems. However, these modulations are attractive for slow frequency-hopping systems. We derive the word error probabilities for slow systems with ideal bit interleaving and binary data modulation.

When no jamming or repeater jamming is present, each bit in a word has the same error probability,  $P_b$ . If the bit errors are independent, the probability of a word error is

$$P_w = \sum_{m=1}^c \binom{c}{m} (1 - P_b)^{c-m} P_b^m \quad (72)$$

For a uniform deployment of interferers, we calculate upper and lower bounds of  $P_w$  by using the equations for the upper and lower bounds of  $P_b$ .

The assumption of independent bit errors is reasonable even for differentially coherent demodulation because the interleaving process ensures that each bit of a data word is transmitted over a different frequency channel.

Assuming independent bit errors, we analyze the effect of partial-band jamming. For a given value of  $\mu$ , the number of jammed channels is approximated by

$$J = \text{int}(\mu M) = \text{int}\left(\frac{\mu M_u w}{c}\right) \quad (73)$$

The probability of a word error can be written as

$$P_w = \sum_{m=1}^c P(m) \quad (74)$$

where  $P(m)$  is the probability of exactly  $m$  bit errors in a word of  $c$  bits. To minimize burst errors due to jamming,  $c$  different channels are used in transmitting a code word. We assume that

$c \leq M$ . Let  $E_n$  denote the event that  $n$  channels out of these  $c$  channels contain jamming power. The probability of  $E_n$  is denoted by  $P(E_n)$ . The probability of exactly  $m$  bit errors given  $E_n$  is denoted by  $P(m|n)$ . From these definitions, it follows that

$$P(m) = \sum_n P(m|n) P(E_n) \quad (75)$$

The summation needs to be carried out only over those values of  $n$  for which  $P(m|n) P(E_n)$  is nonzero. From the definitions, we obtain the following bounds:

$$0 \leq n \leq c; \quad n \leq J; \quad c - n \leq M - J \quad (76)$$

We can evaluate  $P(E_n)$  by considering the  $c$  transmission channels of a word as fixed and randomly choosing the jammed channels. Alternatively, we can consider the jammed channels as fixed and the  $c$  transmission channels as randomly chosen. The two resulting formulas for  $P(E_n)$  can easily be shown to be equal. From elementary combinatorial analysis,

$$P(E_n) = \frac{\binom{c}{n} \binom{M-c}{J-n}}{\binom{M}{J}} = \frac{\binom{J}{n} \binom{M-J}{c-n}}{\binom{M}{c}} \quad (77)$$

Let  $F_i$  denote the event that  $i$  errors occur in those bits that are transmitted in jammed channels. The probability of  $F_i$  given  $E_n$  is denoted by  $P(F_i|E_n)$ . The probability of exactly  $m$  bit errors given the event  $F_i \cap E_n$  is denoted by  $P(m|i)$ . From these definitions, it follows that

$$P(m|n) = \sum_i P(m|i) P(F_i|E_n) \quad (78)$$

The summation needs to be carried out only over those values of  $i$  for which  $P(m|i) P(F_i|E_n)$  is nonzero. From the definitions, we obtain the following bounds:

$$0 \leq i \leq n; \quad i \leq m; \quad m - i \leq c - n \quad (79)$$

From its definition,  $P(m|n,i)$  is equal to the probability that there are  $m - i$  errors among the  $c - n$  bits that are transmitted in channels without jamming power. We assume the independence of bit errors among these  $c - n$  bits. Consequently,

$$P(m|n,i) = \binom{c-n}{m-i} P_{b0}^{m-i} (1 - P_{b0})^{c-n-m+i}, \quad (80)$$

where  $P_{b0}$  is the probability of a bit error when the transmission channel is not jammed. Similarly, we assume the independence of bit errors among the  $n$  bits that are transmitted in jammed channels. If all jammed channels receive the same jamming power and  $i \leq n$ ,

$$P(F_i|E_n) = \binom{n}{i} P_{bi}^i (1 - P_{bi})^{n-i}, \quad (81)$$

where  $P_{bi}$  is the probability of a bit error when the transmission channel is jammed.

Combining the above results, we obtain the word error probability for slow frequency hopping with single-channel, binary data modulation and ideal bit interleaving:

$$P_w = \sum_{m=r}^c \sum_{n=n_0}^{n_1} \sum_{i=i_0}^{i_1} \frac{\binom{J}{n} \binom{M-J}{c-n} \binom{n}{m-i} \binom{n}{i}}{\binom{M}{c}} \times$$

$$P_{b0}^{m-i} P_{bi}^i (1 - P_{b0})^{c-n-m+i} (1 - P_{bi})^{n-i}, \quad (82)$$

where

$$n_1 = \max(0, c + J - M),$$

$$n_1 = \min(c, J),$$

$$i_0 = \max(0, m - c + n),$$

$$i_1 = \min(m, n).$$

The summation limits ensure that the binomial coefficients are well defined.

We can evaluate  $P_{bn}$  for  $n = 0, 1$  by using equations (58) to (61) and (66) with  $P_s(j, k)$  replaced by  $P_{sn}(j, k)$ . In the notation of section 4,  $P_{sn}(j, k)$  is the probability of a bit error given  $B_{jk} \cap D_n$ . A derivation similar to that of equation (65) yields

$$P_{sn}(j, k) =$$

$$\int_0^\infty \dots \int_0^\infty S_n \left( \sum_{i=1}^j x_i + K \sum_{i=1}^k y_i \right) \prod_{i=1}^j f(x_i) \prod_{i=1}^k f(y_i) dx_1 \dots dx_j dy_1 \dots dy_k, \quad (83)$$

where  $S_n(x)$  is the bit error probability given  $D_n$  and an interference-to-signal ratio of  $x$ .

If spectral splatter is ignored, we may set  $K = 0$  in equation (83) to obtain

$$P_n(j) = \int_0^\infty \dots \int_0^\infty S_n \left( \sum_{i=1}^j x_i \right) \prod_{i=1}^j f(x_i) dx_1 \dots dx_j, \quad (84)$$

We can evaluate  $P_{bn}$  for  $n = 0, 1$  by using equations (8), (10) to (12), and (66) with  $P_j$  replaced by  $P_n(j)$ . If the nonuniform deployment statistics of equation (16) are assumed, equation (84) reduces to

$$P_n(j) = \frac{\alpha^j}{(j-1)!} \int_0^\infty S_n(x) x^{j-1} \exp(-\alpha x) dx, \quad j > 1, \quad (85a)$$

$$P_n(0) = S_n(0). \quad (85b)$$

For differentially coherent MSK data modulation,  $S_n(x)$  is given by equations (54) and (55).

For a uniform deployment of interferers, we calculate the upper and lower bounds of equation (82) by using the appropriate equations for the upper and lower bounds of  $P_{b0}$  and  $P_{b1}$ .

Figures 29 to 35 show the word error probability as a function of the number of interferers for slow hopping with bit interleaving. We assume a uniform deployment with  $L = 2$ ,  $R_0/D = 0.2$ , and  $R_1/D = 2.0$ . The data modulation is MSK and spectral splatter is assumed to be negligible. In figures 29 to 33, the uncoded word length is  $w = 4$  and the signal-to-noise ratio per uncoded bit is  $R_s/N_{tu} = 15$  dB. It is convenient to denote the number of equivalent channels before coding by the parameter  $M_{u1} = M_u/d$ .

Figures 29 and 30 illustrate the improvement due to coding when jamming is absent. In figure 29, we have no coding, so that  $c = 4$  and  $r = 1$ . In figure 30, block coding is used with  $c = 7$ ,  $r = 2$ . The improvement due to coding decreases as the number of interferers increases.

Figures 31 to 33 illustrate the degradation in  $P_w$  caused by jamming. We assume block coding with  $c = 7$  and  $r = 2$ . Figure 31 illustrates the effect of weak repeater jamming. Figures 32 and 33 illustrate the effects of moderate and strong partial-band jamming, respectively. In figure 32, the jamming-to-signal ratio in each jammed channel of the coded transmission is  $N_j/R_s = -5$  dB; in figure 33,  $N_j/R_s = 0$  dB.

The use of repetition coding with fast hopping or slow hopping and bit interleaving is often very effective in reducing the error rates. Repetition coding consists of transmitting an odd number of code bits or chips for each data bit. The receiver decides the logical state of the data bit according to the logical states of the majority of the received bits. Since a code word of  $c$  bits is transmitted for each data bit, the probability of a data bit error is equal to  $P_w$  with  $w = 1$  and  $r = (c + 1)/2$ . As an example, we consider the case in which  $M_u = 2000$ ,  $d = 1$ , and no jamming is present. Figures 34 and 35 show the probability of a data bit error as a function of the signal-to-noise ratio per data bit,  $R_s/N_{tu}$ , for  $c = 1, 3, 5$ , and  $7$ . For figure 34, the

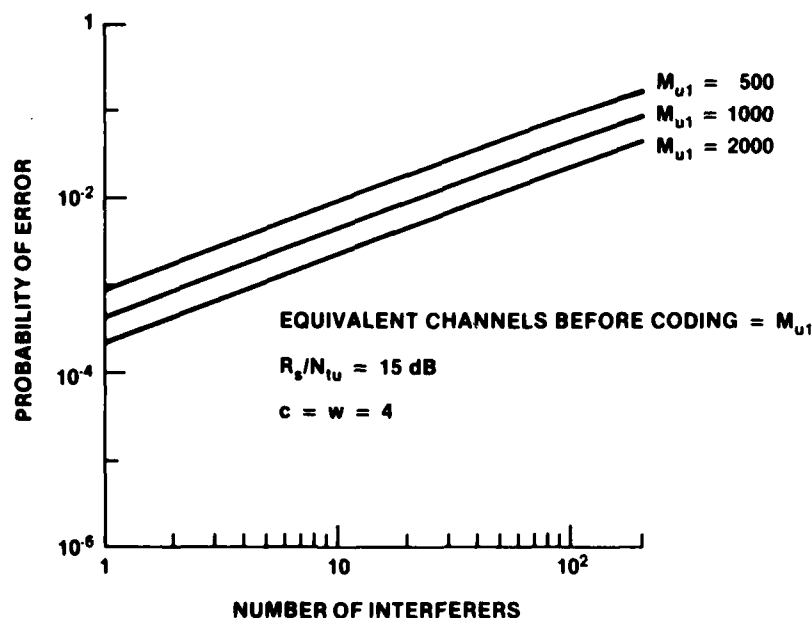


Figure 29. Word error probability for uniform deployment, MSK, no coding, and no jamming.

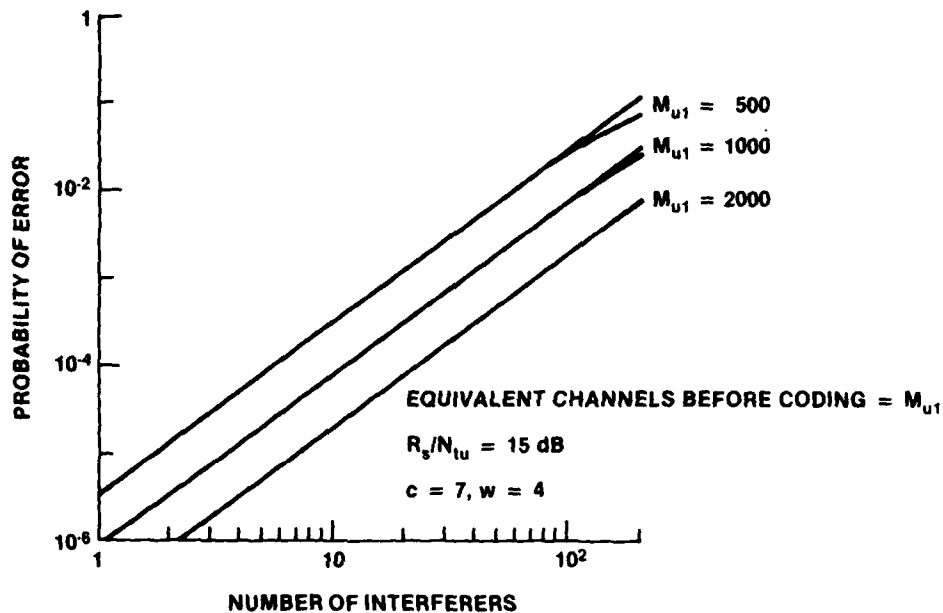


Figure 30. Word error probability for uniform deployment, MSK, block coding, and no jamming.

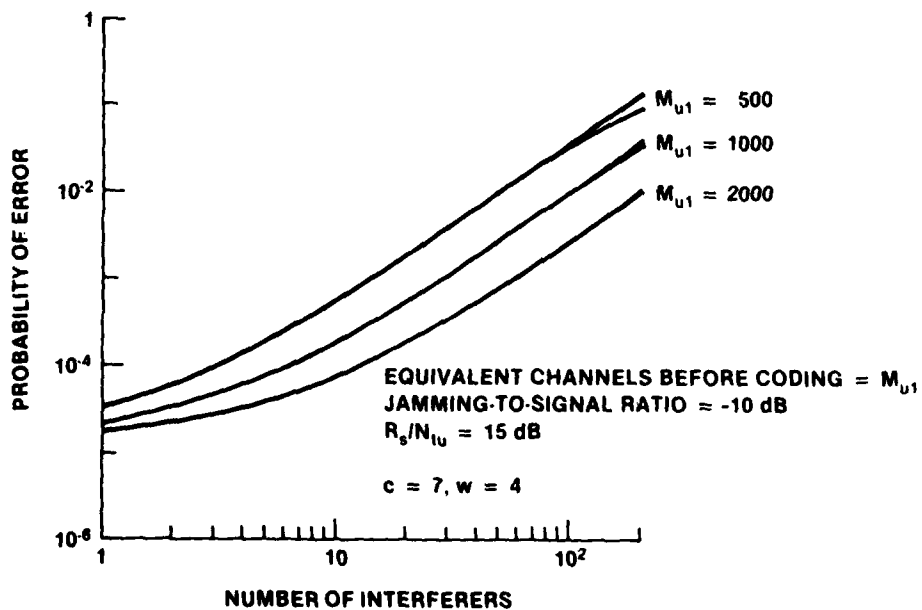


Figure 31. Word error probability for uniform deployment, MSK, block coding, and weak repeater jamming.

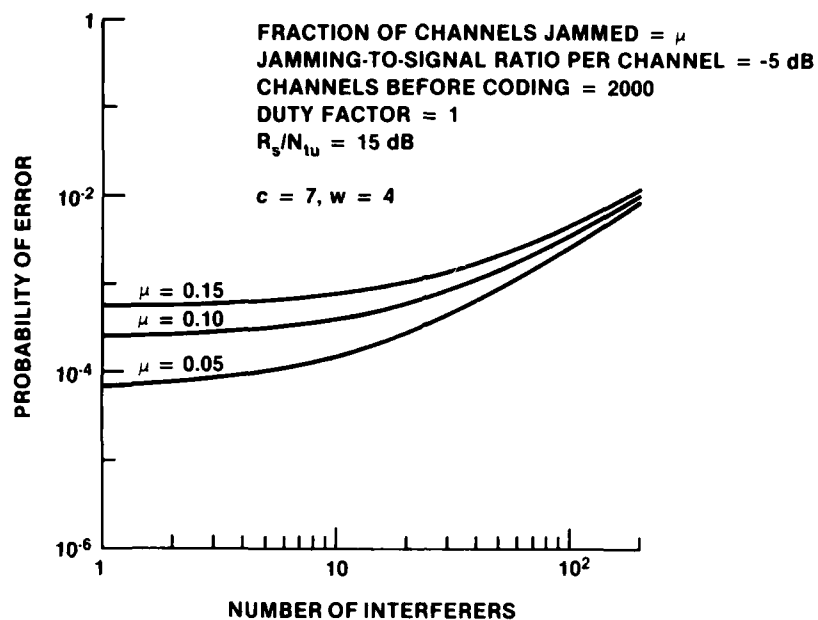


Figure 32. Word error probability for uniform deployment, MSK, block coding, and moderate partial-band jamming.

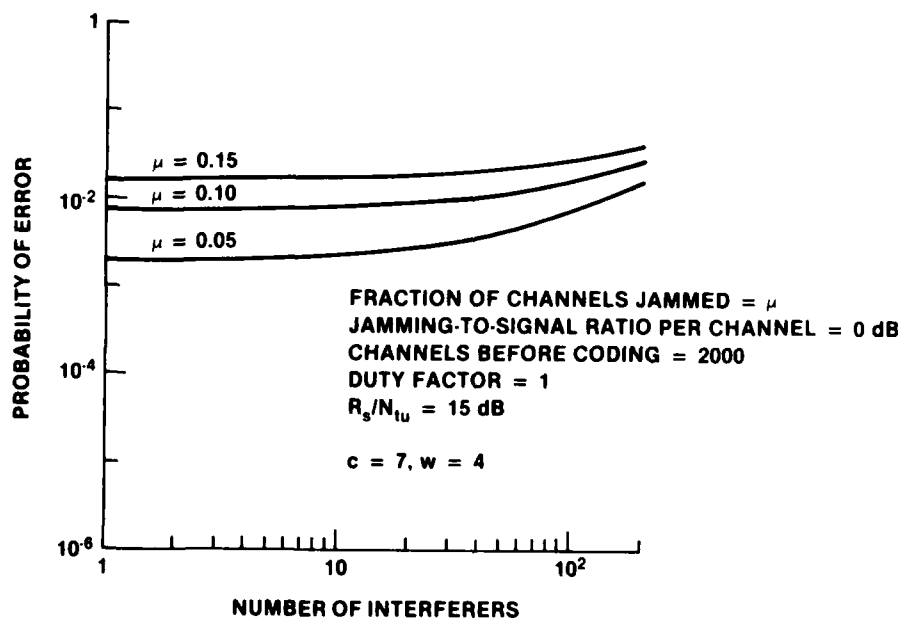


Figure 33. Word error probability for uniform deployment, MSK, block coding, and strong partial-band jamming.



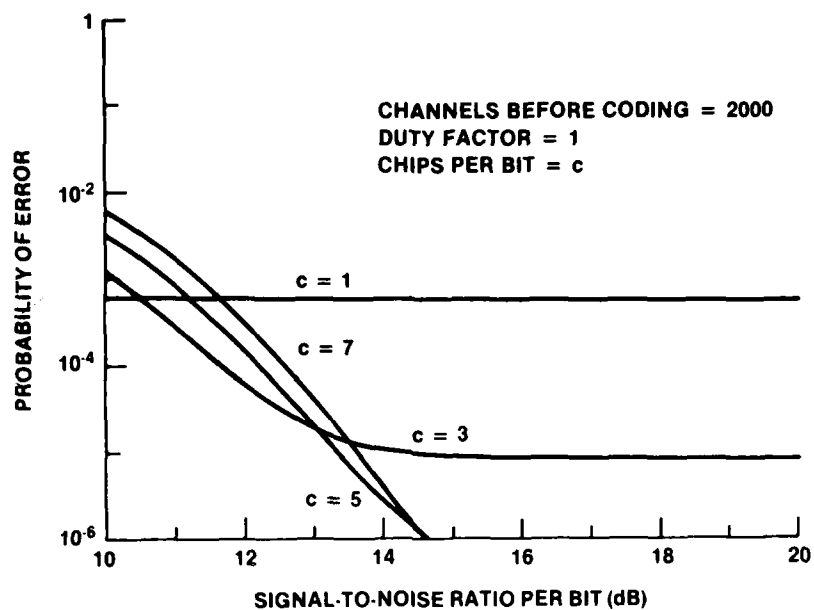


Figure 34. Bit error probability for uniform deployment, MSK, repetition coding, no jamming, and 10 interferers.

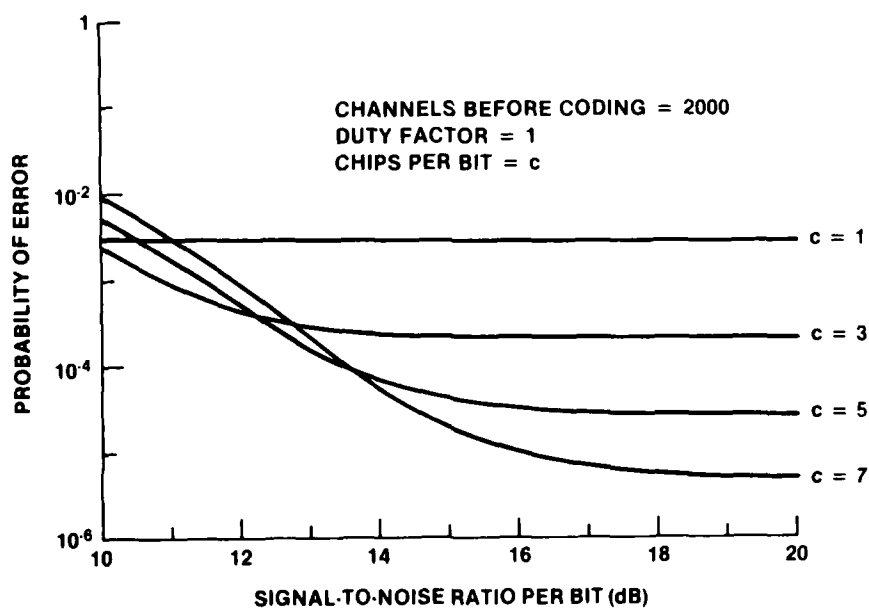


Figure 35. Bit error probability for uniform deployment, MSK, repetition coding, no jamming, and 50 interferers.

number of interferers is  $N = 10$ ; in figure 35,  $N = 50$ . We observe a threshold effect whereby increasing the amount of repetition is helpful only if the received power is sufficiently great.

## 7. CONCLUSIONS

Although the performance of a frequency-hopping network depends upon a host of factors, a few general conclusions can be drawn. The impact of mutual interference on the network is a sensitive function of the number of interferers and the proximity of close interferers that contribute spec-

tral splatter. If there are no close interferers and  $R_0 \geq 0.2D$ , spectral splatter is not an important effect in practical networks with  $K_0 \geq 0.9$  and MSK data modulation. To reduce the susceptibility of a frequency-hopping network to barrage jamming, it is good to have as large a total bandwidth as possible. To reduce the effects of mutual interference, the total bandwidth may be divided into a large number of available hopping channels. However, if the total bandwidth and message characteristics are fixed, increases in the number of channels eventually lead to sufficient spectral splatter to offset any potential performance improvement.

## GLOSSARY OF PRINCIPAL SYMBOLS

$c$	Number of code bits in word
$d$	Duty factor
$D_c$	Distance between two communicators (fig. 2)
$D$	Normalized distance between two communicators (eq (13))
$D_n$	Event that $n$ of the channels associated with a transmission are jammed
$f(u)$	Probability density for interference-to-signal ratio due to single interferer
$g(r)$	Radial probability density for separation between interferer and receiver
$J$	Number of jammed channels
$K$	Adjacent splatter ratio, $K_1/K_0$
$K_0$	Fractional power within transmission channel
$K_i$	Fractional power, due to spectral splatter, intercepted by channel that is $i$ channels removed from transmission channel
$M$	Number of available frequency channels
$M_1$	Equivalent number of available frequency channels, $M/d$
$M_u$	Number of available frequency channels when there is no coding
$M_{u1}$	Equivalent number of available frequency channels when there is no coding, $M_u/d$
$N$	Number of potential interferers
$N_j$	Jamming power
$N_t$	Thermal noise power in channel
$N_{tu}$	Thermal noise power in channel if no coding is used
$P_b$	Probability of bit error
$P_{b0}$	Probability of bit error when transmission channel is not jammed
$P_{b1}$	Probability of bit error when transmission channel is jammed
$P_L$	Lower bound of $P_b$
$P_U$	Upper bound of $P_b$
$P_{jk}$	Probability of bit error given $j$ transmission channel interferers and $k$ complementary channel interferers (FSK)
$P(x_1, \dots, y_k)$	Probability of bit error given that $x_1, \dots, y_k$ are the interference-to-signal ratios in the transmission and complementary channels (FSK)
$P_j$	Probability of bit error given $j$ transmission channel interferers
$P(x_1, \dots, x_j)$	Probability of bit error given that $x_1, \dots, x_j$ are the interference-to-signal ratios in the transmission channel due to mutual interference
$P_s(j, k)$	Probability of bit error given $j$ transmission channel interferers and $k$ adjacent channel interferers
$P(x, y)$	Probability of bit error given that $x$ and $y$ are the interference-to-signal ratios in the transmission and complementary channels, respectively (FSK)
$P(x)$	Probability of bit error given that $x$ is the interference-to-signal ratio in the transmission channel
$P_w$	Probability of word error
$q$	Number of channels, on each side of interferer's transmission channel, in which splatter effects are significant

$R_0$	Minimum separation between interferer and receiver (fig. 2)
$R_1$	Distance from receiver beyond which interferers can be ignored (fig. 2)
$R_s$	Power in desired signal
$S_n(x,y)$	Probability of bit error given $D_n$ and given that $x$ and $y$ are the interference-to-signal ratios in the transmission and complementary channels, respectively (FSK)
$S_n(x)$	Probability of bit error given $D_n$ and given that $x$ is the interference-to-signal ratio in the transmission channel
$w$	Number of bits in a word before coding
$\alpha$	Parameter of nonuniform density for interference-to-signal ratio (eq (18))
$\beta$	Separation at which the nonuniform radial density attains its maximum
$\Gamma$	Number of close interferers that cause significant splatter when hopping in channels beyond the adjacent channels
$\mu$	Fraction of available channels that contain jamming, $J/M$

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